Massive MIMO Optimization Problems for Students

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The documents contain examples of problem statements in the following four topics:

i. Optimal single-user precoding in Massive MIMO communication

ii. Optimal multi-user precoding in Massive MIMO communication

iii. Submodular and DS optimization

iv. Optimal multi-user scheduling

Problems of Topic I and Topic II correspond to multidimensional non-convex optimization problems, in particular optimization on a product of multidimensional complex spheres.

Problems of Topic III and Topic IV correspond to combinatorial optimization, in particular set-function optimization with knapsack constraints.

Topic I

Optimal single-user precoding in Massive MIMO communication

Motivation:

Modern Base Stations may have antenna arrays consisted of 32, 64 or 128 antennas, and User Equipment (e.g. smartphone) may have up to 4 or 8 antennas. All the antennas transmit on the same frequency and in the same time. To manage interference Base Station applies different phase shifts on different antennas so that interference picture has its maximum on the User Equipment. Precoding is actually the selection of these phase shifts.

Moreover, Base Station may send several streams of information to the particular user multiplexing its download speed. This streams should be precoded in the proper way to maximize performance. The particular optimization problem depends on the type of post-processing algorithm adopted by User Equipment.

Let

Rank – number of user streams, not greater than number of UE antennas,

$\boldsymbol{H}$ – Channel matrix, that is R (number of UE antennas) by T (number of BS antennas) matrix of complex numbers, each number has modulus not greater than 1,

$\boldsymbol{P}, \boldsymbol{\delta^2}$ – total signal power and noise power, real positive values,

$\boldsymbol{W}$ – matrix of Precoding, T (number of BS antennas) by Rank, each column has Euclidian norm 1.

$\boldsymbol{sinr_k}$ – signal-to-interference-and-noise ratio of stream k,

We are interested in maximization of total spectral efficiency, which is defined by formula:

$$F = \sum_{k=1,..,Rank} \log(1 + sinr_k)$$

Problem 1.1 (Precoding in case of linear post-processing)

Assume linear post-processing method is used on the receiver. Than a matrix G is applied to the signals received by receiver antennas.

In this case $sinr_k$ can be estimated as follows (power is equally distributed between the streams):

$$sinr_k = \frac{\left|[G \cdot H \cdot W]_{k,k}\right|^2 \frac{P}{Rank}}{\sum_{i \neq k}\left|[G \cdot H \cdot W]_{k,i}\right|^2 \frac{P}{Rank} + \delta^2 [G \cdot G^*]_{k,k}}$$

Where G is defined as

a) In case of zero-forcing post-processing: $G = ((HW)^* HW)^{-1} (HW)^*$,

b) In case of linear MMSE post-processing: $G = \left((HW)^* HW + \frac{Rank \cdot \delta^2}{P} I\right)^{-1} (HW)^*$.

   See [1] for the proof of this formula.

c) In case of IRC (Interference Rejection Combining) post-processing: see [2]

Given $Rank \in [1, \dots R]$, H, P, δ^2, G, find the precoding matrix W that provides maximum for the functional:

$$F(W) = \sum_{k=1,..,Rank} \log(1 + sinr_k(W)) \rightarrow \max_W$$

Problem 1.2 (Case of maximum likelihood post-processing)

See [3] for low-cost sinr estimation in the case of **maximum likelihood post-processing.** Given $Rank \in [1, \dots R]$, H, P, δ^2, find the precoding matrix W that provides maximum for the total spectral functional F in this case.

Problem 1.3 (Optimal power allocation)

Find optimal power allocation $p_1, \dots, p_{Rank}$, $\sum_{k=1,..,Rank} p_k = P$ for total spectral efficiency functional:

$$F(p_1, \dots, p_{Rank}) = \sum_{k=1,..,Rank} \log(1 + sinr_k(p_1, \dots, p_{Rank})) \to \max_{p_1,\dots,p_{Rank}}$$

Assuming one of the post-processing algorithm and optimal precoding.

Problem 1.4 (Capacity maximization)

Alternative approach is to optimize channel capacity instead of the total spectral efficiency. See [4] for definition of MIMO channel capacity and the proof of its formula.

References:

[1] Eraslan, Eren, Babak Daneshrad, and Chung-Yu Lou. "Performance indicator for MIMO MMSE receivers in the presence of channel estimation error." IEEE Wireless Communications Letters 2, no. 2 (2013): 211-214.

[2] Cheng, Chien-Chun, Serdar Sezginer, Hikmet Sari, and Yu T. Su. "SINR Enhancement of Interference Rejection Combining for the MIMO Interference Channel." In 2014 IEEE 79th Vehicular Technology Conference (VTC Spring), pp. 1-5. IEEE, 2014.

[3] Redlich, Oded, Doron Ezri, and Dov Wulich. "SNR estimation in maximum likelihood decoded spatial multiplexing." *arXiv preprint arXiv:0909.1209* (2009).

[4] Holter, Bengt. "On the capacity of the MIMO channel: A tutorial introduction." In Proc. IEEE Norwegian Symposium on Signal Processing, pp. 167-172. 2001.

Topic II

Optimal multi-user precoding in Massive MIMO communication

Massive MIMO technology allows to transmit different streams to different users, serving several users at same frequency in the same time.

Assume for simplicity that each of the served users has only one stream.

$\boldsymbol{K} \in \mathbb{N}$ – fixed number of served users (maximum 32, typically not larger than 16)

$\boldsymbol{h_k} \in \mathbb{C}^{Tx}, k = 1, \dots, K, \left|h_k^i\right| \le 1$ – given channel vectors (each vector dimension is equal to number of base station antennas),

$\boldsymbol{\alpha_k} \in \mathbb{R}^{+}, k = 1, \dots, K$ – given user priorities,

$\boldsymbol{sinr_k} \in \mathbb{R}^{+}, k = 1, \dots, K$ – given single-user signal-to-noise ratio estimation. In case of multi-user transmission, each user $sinr_k$ may become different.

$\boldsymbol{\omega_k} \in \mathbb{C}^{Tx}, k = 1, \dots, K, \left|\left|\omega_k\right|\right|_2 = 1$ – precoding vectors.

Problem 2.1 (Single stream per user, equal power distribution)

We are interested in Utility maximization that has the following form:

$$F(\omega_1, \omega_2, \dots, \omega_K) = \sum_{k=1}^{K} \alpha_k \cdot \log\left(1 + \frac{sinr_k \cdot \| < \omega_k, h_k > \|^2}{K + sinr_k \cdot \sum_{i \ne k} \| < \omega_i, h_k > \|^2}\right) \to \max_{\{\omega\}}$$

1. For a classic ZF (zero-forcing) approach see [5]
2. Propose a solution for this problem (maybe in the form of algorithm) that has the similar computational complexity with ZF, but provides better result for Utility in average or in some domain.

   See also [6] for the related research.

Problem 2.2 (Single stream per user, general power distribution)

1. What is the optimal power allocation between the users in the case of ZF precoding?
2. Propose a low-cost solution for the joint power and precoding Utility optimization:

$$F(\omega_1, \dots, \omega_K, p_1, \dots, p_K) = \sum_{k=1}^{K} \alpha_k \cdot \log\left(1 + \frac{p_k \cdot sinr_k \cdot \| < \omega_k, h_k > \|^2}{1 + sinr_k \cdot \sum_{i \ne k} p_i \cdot \| < \omega_i, h_k > \|^2}\right) \to \max_{\{\omega, p\}},$$

$$\sum_{k=1}^{K} p_k = 1.$$

References:

[5] Wiesel, Ami, Yonina C. Eldar, and Shlomo Shamai. "Optimal generalized inverses for zero forcing precoding." In *2007 41st Annual Conference on Information Sciences and*

Systems, pp. 130-134. IEEE, 2007.

[6] Björnson, Emil, Mats Bengtsson, and Björn Ottersten. "Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure [lecture notes]." *IEEE Signal Processing Magazine* 31, no. 4 (2014): 142-148.

Topic III

Submodular and DS optimization

Consider ground set S.

- Set-function $\mathrm{F}: 2^{\mathrm{S}} \to \mathrm{R}$ is called **sub-modular** iff for any $A \subseteq \mathrm{B} \subseteq \mathrm{S}$, $c \in \mathrm{S} \setminus \mathrm{B}$ implies $\mathrm{F}(\mathrm{A} \cup \mathrm{c}) - \mathrm{F}(\mathrm{A}) \geq \mathrm{F}(\mathrm{B} \cup \mathrm{c}) - \mathrm{F}(\mathrm{B})$. This property is an analogue of concavity for real-valued functions.
- Set-function $\mathrm{F}: 2^{\mathrm{S}} \to \mathrm{R}$ is called **super-modular** iff for any $A \subseteq \mathrm{B} \subseteq \mathrm{S}$, $c \in \mathrm{S} \setminus \mathrm{B}$ implies $\mathrm{F}(\mathrm{A} \cup \mathrm{c}) - \mathrm{F}(\mathrm{A}) \leq \mathrm{F}(\mathrm{B} \cup \mathrm{c}) - \mathrm{F}(\mathrm{B})$. This property is an analogue of convexity for real-valued functions.
- Set-function $\mathrm{F}: 2^{\mathrm{S}} \to \mathrm{R}$ is called **modular** iff it is both sub-modular and super-modular. Modular set-function is an analogue of linear real-valued function.
- Set-function $\mathrm{F}: 2^{\mathrm{S}} \to \mathrm{R}$ is called **monotone** *iff* for any $A \subseteq B \subseteq S$, $F(A) \leq F(B)$.
- Set-function is **DS** iff it can be represented as a **difference of two monotone** submodular function. In is an analogue of DC real-valued functions.

Papers on non-monotone sub-modular optimization: [7], [8], [9], [10], [11], [12]

Paper on DS optimization: [13]

Problem 3.1 (One-pass approach)

Consider the following problem:

$$F(A) \to \max_{A \subseteq S},$$

$$|A| \leq K.$$

One pass approach to the optimization is describes as follows:

- Initialize $A := \emptyset$.

- Sort users in the decreasing order w.r.t. $F(\{a\})$.
- Look through the users and for each next user n check the condition:

$$\Delta F_n(A) \coloneqq F(A \cup n) - F(A) > 0$$

    - If it is satisfied, add user n to A, else skip it.
- When $|A| = K$ or all the users were checked, output the set A.

Is it possible to describe a class of set-functions for which one-pass approach provides a good optimum approximation (with a guaranteed approximation accuracy)?

Problem 3.2 (Low-cost optimization of non-monotone sub-modular function and DS-function)

Let us define complexity of an evaluation F(A) as |A|. Complexity of optimization method can be defined as a sum of complexity of all the required evaluations.

Consider the size of ground set |S|=30. Assume F belongs to one of the following classes:

I. Sub-modular but not monotone function,

II. DS function.

Propose an optimization method (with a guaranteed accuracy of optimum approximation) that has complexity not greater than:

a) 150% of the one-pass approach complexity,

b) 300% of the one-pass approach complexity,

c) 1000% of the one-pass approach complexity.

References:

[7] Feige, Uriel, Vahab S. Mirrokni, and Jan Vondrak. "Maximizing non-monotone submodular functions." SIAM Journal on Computing 40, no. 4 (2011): 1133-1153.

[8] Badanidiyuru, Ashwinkumar, and Jan Vondrák. "Fast algorithms for maximizing submodular functions." In *Proceedings of the twenty-fifth annual ACM-SIAM symposium on Discrete algorithms*, pp. 1497-1514. Society for Industrial and Applied Mathematics, 2014.

[9] Mirzasoleiman, Baharan, Ashwinkumar Badanidiyuru, Amin Karbasi, Jan Vondrák, and

Andreas Krause. "Lazier than lazy greedy." In *Twenty-Ninth AAAI Conference on Artificial Intelligence*. 2015.

[10] Fahrbach, Matthew, Vahab Mirrokni, and Morteza Zadimoghaddam. "Non-monotone submodular maximization with nearly optimal adaptivity complexity." arXiv preprint arXiv:1808.06932 (2018).

[11] Gillenwater, Jennifer. "Maximization of non-monotone submodular functions." (2014).

[12] Buchbinder, Niv, Moran Feldman, Joseph Seffi Naor, and Roy Schwartz. "Submodular maximization with cardinality constraints." In Proceedings of the twenty-fifth annual ACM-SIAM symposium on Discrete algorithms, pp. 1433-1452. Society for Industrial and Applied Mathematics, 2014.

[13] Iyer, Rishabh, and Jeff Bilmes. "Algorithms for approximate minimization of the difference between submodular functions, with applications." arXiv preprint arXiv:1207.0560 (2012).

Topic IV

Optimal multi-user scheduling

Assume we have set U of active users at some particular time moment, $|\mathrm{U}| = \mathrm{N}$. We know precoding selection algorithm and power allocation policy.

The frequency band is divided into several (about 15) recourse blocks. For each resource block the scheduler needs to select a subset of users to allocate. On different recourse blocks the sets of allocated users may be different.

For each user $k = 1, \dots, N$ we know

- $\boldsymbol{\alpha_k} \in \mathbb{R}^+$– user priority,
- $\boldsymbol{sinr_k} \in \mathbb{R}^+$ – single-user signal-to-noise ratio estimation,
- $\boldsymbol{H_k^{RB}} \in \mathbb{C}^{Rx \times Tx}$, $\left|[H_k]_{ij}\right| \leq 1$ – given channel matrix, specific for resource block (RB)

At each particular resource block the scheduling problem is formulated as follows:

$$F(A) = \sum_{k \in A} \mathrm{\alpha_k} \cdot \mathrm{SpecEff}(\{H_i\}_{i \in A}, sinr_k) \to \max_{A \subseteq U},$$

$$\sum_{k \in A} rank_k \leq K,$$

where the particular form of multi-user spectral efficiency estimation $\mathrm{SpecEff}(\{H_i\}_{i\in A}, sinr_k)$ depends on the precoding selection algorithm and power allocation policy. For standard precoding methods its formula and the proof can be found in [14]. Also it is proved that the functional is a DS set-function.

Disclaimer*: the following two problems have not only theoretical, but also an experimental aspect. The data set of channel matrix realizations will be provided. The result is expected to be verified on this set.*

Problem 4.1 (Submodular approximation)

Provide a submodular approximation for the set-function $F(\cdot)$, such that its optimum is close to the optimum of the original set-function.

Problem 4.2 (Multi-user rank adaptation)

For each user k its $rank_k$ should be the same on all the resource blocks where it is allocated. Propose a low-cost heuristic for $rank_k$ selection.

References:

[14] Ghasempour, Yasaman, Narayan Prasad, Mohammad Khojastepour, and Sampath Rangarajan. "Novel combinatorial results on downlink mu-mimo scheduling with applications." In *2017 13th Annual Conference on Wireless On-demand Network Systems and Services (WONS)*, pp. 152-159. IEEE, 2017.