Massive MIMO

Introduction

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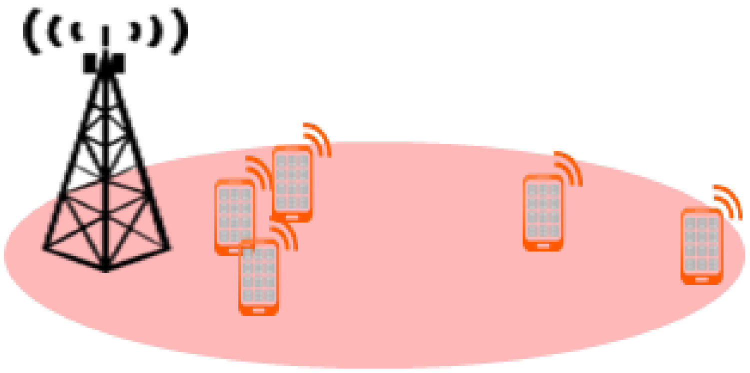
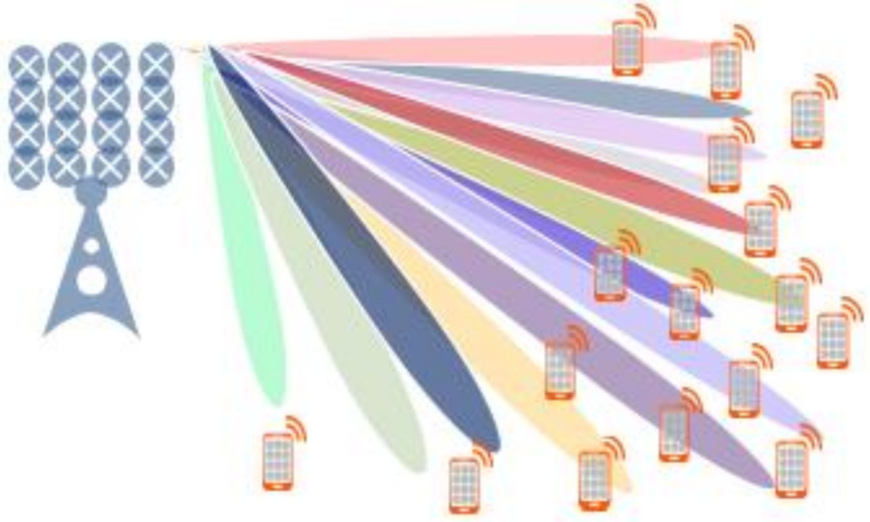
Single antenna transmission



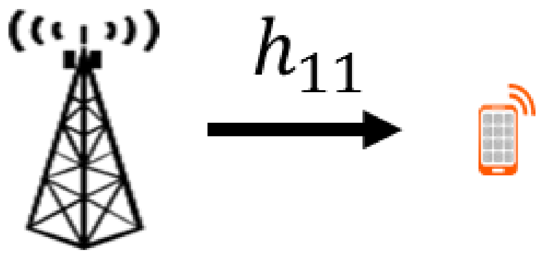
Massive MIMO transmission – key 5G technology



Source of

- Non-convex optimization problems
- Combinatorial optimization problems
- Stochastic optimization problems
- Dynamic control problems

Single antenna transmission



After modulation data is represented by a complex number (symbol):

$$x_1 \in \mathbb{C}$$

$$|x_1|^2 = 1$$

Channel h_{ij}:

$$h_{ij} \in \mathbb{C},\ |h_{ij}| \leq 1$$

$$i \rightarrow j$$

Transmitter antenna | Receiver antenna

Describes signal transformation during transmission from antenna i to antenna j

Transmission with power p

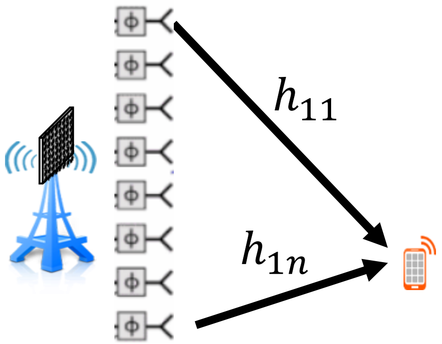
$$y_1 = h_{11} \cdot p \cdot x_1 + noise$$

Received symbol | Transmitted symbol

$$SINR_1 = \frac{P(signal)}{P(noise + interference)} = \frac{p \cdot |h_{11}|^2}{\delta^2}$$

Shannon theorem:
upper bound for information transmission capacity is $\log(1 + SINR)$

Multi antenna transmission



n – number of transmitting antennas

w_k^1– “**weight**” of the symbol at antenna

Symbol x_1 is multiplied by w_k^1 and then transmitted from k-th antenna

$$w^1 = \begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix}$$ -- precoding vector

$w^1 \in \mathbb{C}^n$,

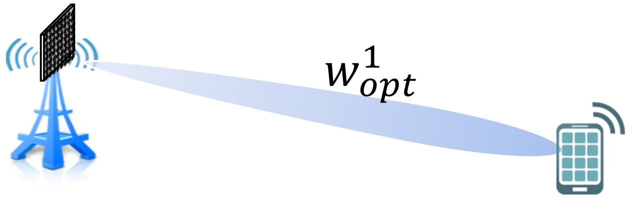
$\left\| w^1 \right\|^2_{L^2} = p$,

$$y_1 = (h_{11} \ldots h_{1n}) \cdot \begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \cdot x_1 + noise$$

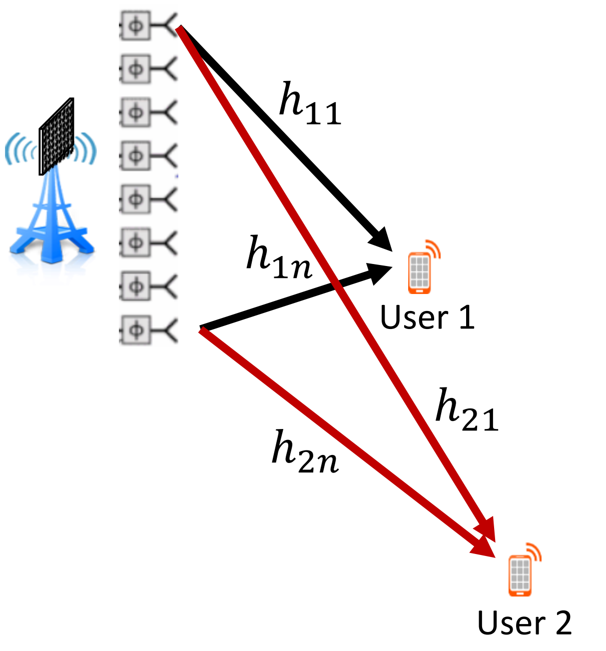
$$SINR_1 = \frac{\left|\langle h_1, {w^1}^* \rangle\right|^2}{\delta^2}$$

Which **precoding vector** w^1 maximizes $SINR$ for user with channel h_1?

Answer: $w_{opt}^1 = c \cdot h_1^*$



Multi-user transmission



Transmit two symbols to two different users simultaneously

$$y_1 = (h_{11} \ldots h_{1n}) \cdot \left(\begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \cdot x_1 + \begin{pmatrix} w_1^2 \\ \vdots \\ w_n^2 \end{pmatrix} \cdot x_2 \right) + noise_1$$

$$y_2 = (h_{21} \ldots h_{2n}) \cdot \left(\begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \cdot x_1 + \begin{pmatrix} w_1^2 \\ \vdots \\ w_n^2 \end{pmatrix} \cdot x_2 \right) + noise_2$$

$$w^k = \begin{pmatrix} w_1^k \\ \vdots \\ w_n^k \end{pmatrix}$$

$w^k \in \mathbb{C}^n$,

$\sum_k \left\| w^k \right\|_{L^2}^2 = p$,

$$y = H \cdot W \cdot x + noise$$

Channel matrix **Precoding matrix**

$$SINR_1(W) = \frac{\left|\langle h_1, w^{1^*} \rangle\right|^2}{\left|\langle h_1, w^{2^*} \rangle\right|^2 + \delta_1^2}$$

$$SINR_2(W) = \frac{\left|\langle h_2, w^{1^*} \rangle\right|^2}{\left|\langle h_2, w^{2^*} \rangle\right|^2 + \delta_2^2}$$

How to choose precoding matrix?

Maximizing weighted sum of spectral efficiency:

$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k) \to \max_W$$

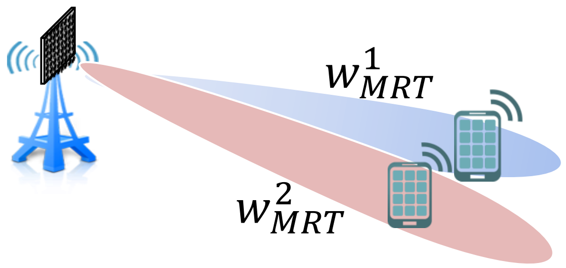
Multi-user beamforming

$$\sum_{u\in U} \alpha_u \cdot \log(1 + SINR_u(W)) \to \max_W$$

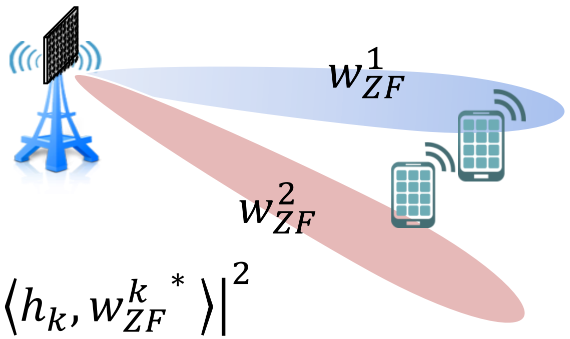
$$SINR_k(W) = \frac{\left|\langle h_k, w^{k^*} \rangle\right|^2}{\left|\langle h_k, w^{l^*} \rangle\right|^2 + \delta_1^2}$$

Classic solution #1: Maximum-rate transmission

$$w_{MRT}^k = c_k \cdot h_k^*$$



Classic solution #2: Zero Forcing



$$SINR_k(W_{ZF}) = \frac{\left|\langle h_k, w_{ZF}^{k}{}^* \rangle\right|^2}{\delta_1^2}$$

ZF beam orthogonal to all other users channel vectors:

$w_{ZF}^k \in < h_1, \dots, h_{k-1,} h_{k+1}, \dots, h_n >^{\perp}$

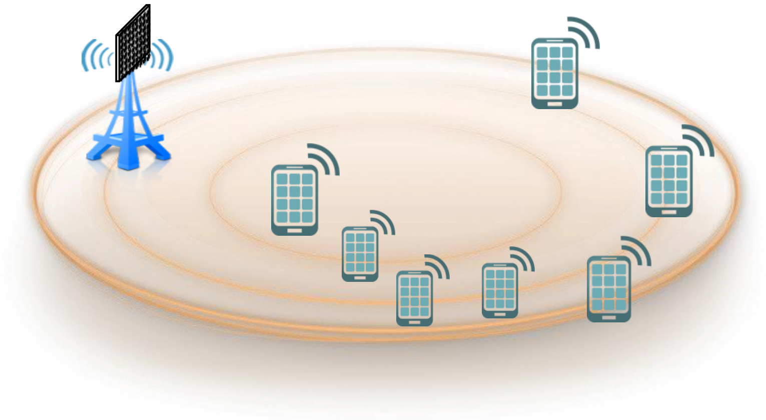
w_{ZF}^k maximizes $\left|\langle h_k, w_{ZF}^{k}{}^* \rangle\right|^2$ in this subspace

$\mathrm{W_{ZF}}$ is a pseudo-inverse matrix to H:

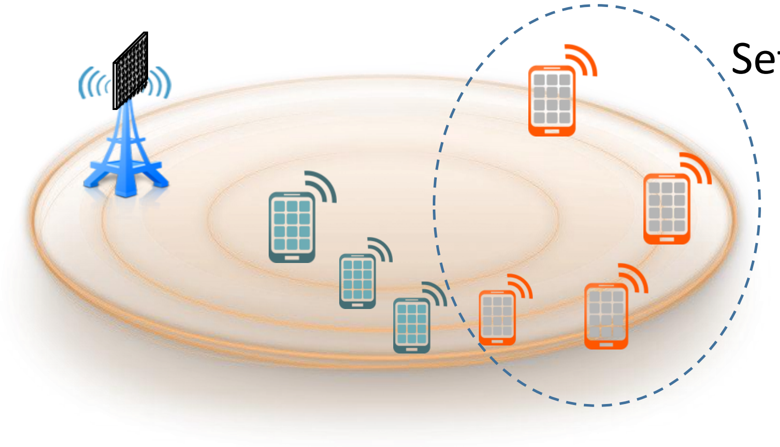
$\mathrm{W_{ZF}} = H^* \cdot (HH^*)^{-1}$

Multi-user pairing

Not necessary to transmit to all active users A

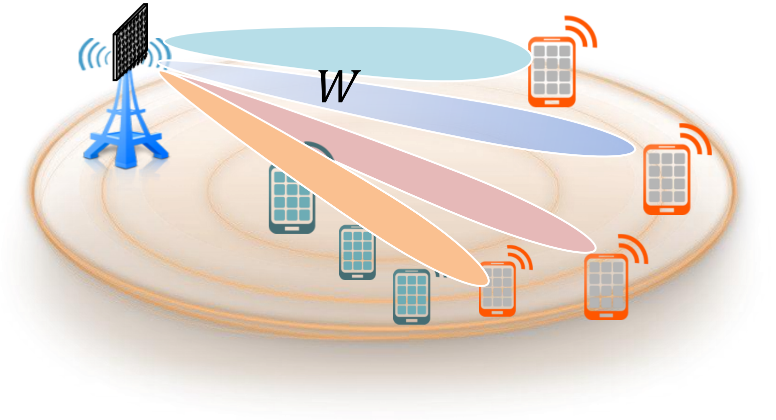


$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k) \to \max_{U \subseteq A}$$



Set $U \subseteq A$

$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k) \to \max_{W}$$



How to select optimal subset $U \subseteq A$ of users for transmission?

$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k(W)) \to \max_{\substack{U \subseteq A \\ W}}$$

Submodular set-function

"The larger the set the smaller the gain"

$F: 2^A \rightarrow \boldsymbol{R}$ *is called* ***submodular*** *iff*
for any $U \subseteq V \subseteq A$, $c \in A \backslash \mathrm{B}$ *implies* $F(U \cup c) - F(U) \geq F(V \cup c) - F(V)$
This property is an analogue for **concavity**

If a function both submodular
and Supermodular
it is called **modular**:
$F(U \cup c) - F(U) = F(V \cup c) - F(V)$
This is the analog of linearity

Supermodular set-function

"The larger the set the larger the gain"

$F: 2^A \rightarrow \boldsymbol{R}$ *is called* **supermodular** *iff*
for any $U \subseteq V \subseteq A$, $c \in S \backslash \mathrm{V}$ *implies* $F(U \cup c) - F(U) \leq F(B \cup c) - F(V)$
This property is an analogue for **convexity**

Set-function is **DS** iff it can be
represented as a
difference of
two monotone
submodular function

Monotone set-function

$F: 2^A \rightarrow \boldsymbol{R}$ *is called* ***monotone*** *iff*
for any $U \subseteq V \subseteq A$, $F(U) \leq F(V)$

$$F(U) = \sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k)$$

Set-function property	Real function analogue property	$F(U)$
Monotone	Monotonicity	no
Modular	Linear	no
Submodular	Concave	no
Supermodular	Convex	no
Difference of two submodular functions	Difference of two concave functions	yes

User pairing from combinatorial point of view

Consider the size of ground set |A|=30. Assume F belongs to one of the following classes:
I. Sub-modular but not monotone function,
II. DS function.

Propose an optimization method (with guaranteed accuracy of optimum approximation) that has complexity not greater than:
a) 150% of the one-pass approach complexity,
b) 300% of the one-pass approach complexity,
c) 1000% of the one-pass approach complexity.

Multi-user beamforming

How to choose precoding matrix W maximizing

$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k) \rightarrow \max_W$$

Single-user beamforming

How to do beamforming if user equipment has several antennas?

How to optimize beamforming for a specific algorithm at receiver end?

User scheduling

How to design coefficients α_k

$$F(U) = \sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k)$$

targeting particular network KPIs?

Thanks