Hamiltonian mechanics
self–preparation sheet for the midterm test

October 9, 2019

1. Determine the equation of the curve giving the shortest distance between two points on the surface of a cone. Let $r^2 = x^2 + y^2$ and $z = r \cot \alpha$.

   Answer 1 *$\theta = \alpha$ and $r \sin\alpha = c_2/\cos(\phi \sin\alpha + c_1)$.*

2. Use Euler-Lagnrage equations to describe the motion with the following Lagrangian
$$L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{(x^2+y^2)^{1/2}}.$$

   Answer 2 *$\ddot{x} = -x/(x^2+y^2)^3$, $\ddot{y} = -y/(x^2+y^2)^3$.*

3. Consider a disk of radius R rolling down an inclined plane of length l and angle α. Find the equations of motion, the angular acceleration, and the force of constraint.

   Answer 3 *see the answer in the next exercise.*

4. Find the Hamiltonian for the following Lagrangian
$$L = \frac{1}{2}m\dot{y}^2 + \frac{1}{4}mR^2\dot{\theta}^2 + mg(y-l)\sin\alpha.$$
   here m, g, l, α are fixed parameters.

   Answer 4 *$H = \frac{p_y^2}{2m} + \frac{p_\theta^2}{mR^2} - Mg(y-R)\sin\alpha$*

5. Write the Langranian of spherical pendulum (mass m ball on the length l rod in $\mathbb{R}^3$) in two different coordinate systems and determine cycles coordinates as well as associated symmetry.

   Answer 5 *In Decartes coordinates $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - mgz$ and in spherical coordinates $L = \frac{1}{2}ml^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right) - mgl\cos\theta$.*

6. For any two vector fields X, Y on a manifold M show that the commutator of the corresponding Lie derivatives satisfies $[L_X, L_Y] = L_{[X,Y]}$.