Affine Screening Operators, Affine Laumon Spaces, and Conjectures Concerning Non-Stationary Ruijsenaars Functions

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Based on the screened vertex operators associated with the affine screening operators, we introduce the formal power series $f^{\widehat{gl}_N}(x,p|s,\kappa|q,t)$ which we call the *non-stationary Ruijsenaars function*. We identify it with the generating function for the Euler characteristics of the affine Laumon spaces. When the parameters s and κ are suitably chosen, the limit $t\to q$ of $f^{\widehat{gl}_N}(x,p|s,\kappa|q,q/t)$ gives us the dominant integrable characters of $\widehat{sl}_N$ multiplied by $1/(p^N;p^N)_\infty$ (*i.e.* the $\widehat{gl}_1$ character). Several conjectures are presented for $f^{\widehat{gl}_N}(x,p|s,\kappa|q,t)$, including the bispectral and the Poincaré dualities, and the evaluation formula. The main conjecture asserts that (i) one can normalize $f^{\widehat{gl}_N}(x,p|s,\kappa|q,t)$ in such a way that the limit $\kappa\to 1$ exists, and (ii) the limit $f^{\mathrm{st.}\,\widehat{gl}_N}(x,p|s|q,t)$ gives us the eigenfunction of the elliptic Ruijsenaars operator. The non-stationary affine q-difference Toda operator $\mathcal{T}^{\widehat{gl}_N}(\kappa)$ is introduced, which comes as an outcome of the study of the Poincaré duality conjecture in the affine Toda limit $t\to 0$. The main conjecture is examined also in the limiting cases of the affine q-difference Toda ($t\to 0$), and the elliptic Calogero-Sutherland ($q,t\to 1$) equations.