

<b>Course Title (in English)</b>	Virasoro Algebra and Conformal Field Theory
<b>Course Title (in Russian)</b>	Алгебра Вирасоро и конформная теория поля
<b>Lead Instructor(s)</b>	Bershtein, Mikhail
<b>Status of this Syllabus</b>	The syllabus is a final draft waiting for form approval
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## 1. Annotation

<b>Course Description</b>	<p>Conformal field theory is a quantum field theory that is invariant under conformal transformations. The course is devoted to a two-dimensional theory, which possess an infinite dimensional algebra of local conformal transformation, which includes Virasoro Lie algebra.</p> <p>We will mainly focus on the mathematical aspects of the theory based on the relations with the representation theory of Virasoro algebra. A small preliminary acquaintance with string theory and conformal field theory is assumed.</p>
<b>Course Prerequisites</b>	Students should be familiar with <ol style="list-style-type: none"><li>1. representation theory of finite dimensional Lie algebras</li><li>2. Complex analysis (one dimensional)</li><li>3. Bosonic string in d dimensions.</li></ol>

## 2. Structure and Content

<b>Course Academic Level</b>	Master-level course suitable for PhD students
<b>Number of ECTS credits</b>	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Bootstrap in 2d CFT	Space of fields in CFT. OPE, stress-energy tensor. Verma modules, Shapovalov form, singular vectors. Asymptotic behaviour of the Shapovalov form.	1	2	2
Virasoro algebra	Vertex operators, conformal blocks. Zamolodchikov's recursion relations.	2	4	4
Degenerate fields	Differential equation on conformal blocks, hypergeometric function. Three point functions	2	4	4
Screening operators.	Definition of the screening operators. Choice of the contour. Formula for the correlation function. Formula for singular vectors. Homology with coefficients in local system.	2	4	4
Coset construction.	Affine lie algebras. Coset construction. Unitary representations of the Virasoro algebra.	1	2	2
Drinfeld-Sokolov reduction.	Classical Drinfeld-Sokolov reduction. Quantum Drinfeld-Sokolov reduction.	1	2	2
AGT relation	AGT relation for Heisenberg algebra. AGT relation for Virasoro algebra.	1	2	2

### 3. Assignments

Assignment Type	Assignment Summary
Problem Set	List of problems and exercises, after each lecture

### 4. Grading

<b>Type of Assessment</b>	Graded	
<b>Grade Structure</b>	<b>Activity Type</b>	<b>Activity weight, %</b>
	Class participation	50
	Problem Set	50
<b>A:</b>	Grading Scale	
<b>B:</b>	86	
<b>C:</b>	76	
<b>D:</b>	66	
<b>E:</b>	56	
<b>F:</b>	46	

## 5. Basic Information

<b>Attendance Requirements</b>	Mandatory with Exceptions	
<b>Course Stream</b>	Science, Technology and Engineering (STE)	
<b>Course Delivery Frequency</b>	Other	
<b>Students of Which Programs do You Recommend to Consider this Course as an Elective?</b>	<b>Masters Programs</b>	<b>PhD Programs</b>
	Mathematical and Theoretical Physics	Mathematics and Mechanics
<b>Course Tags</b>	Math Physics	

## 6. Textbooks and Internet Resources

<b>Required Textbooks</b>	<b>ISBN-13 (or ISBN-10)</b>
Francesco, Philippe, Mathieu, Pierre, Sénéchal, David Conformal Field Theory 1997	978-1-4612-2256-9
Iohara, Kenji, Koga, Yoshiyuki Representation Theory of the Virasoro Algebra	978-0-85729-159-2

<b>Papers</b>	<b>DOI or URL</b>
V.A. Alba, V.A. Fateev, A.V. Litvinov, G.M. Tarnopolskiy, On Combinatorial Expansion of the Conformal Blocks Arising from AGT Conjecture,	10.1007/s11005-011-0503-z
B. Feigin, E. Frenkel, Quantization of the Drinfeld-Sokolov reduction,	10.1016/0370-2693(90)91310-8
P. Goddard, A. Kent and D. Olive Unitary representations of the Virasoro and super-Virasoro algebra	10.1007/BF01464283
A.A. Belavin, A.M. Polyakov, A.B. Zamolodchikov, Infinite conformal symmetry in two-dimensional quantum field theory	10.1016/0550-3213(84)90052-X

## 7. Facilities

## 8. Learning Outcomes

<b>Knowledge</b>
Basic properties of the space of fields in two dimensional conformal field theory. Basic properties of the correlation functions in two dimensional conformal field theory. Basic results in representation theory of the Virasoro algebra
<b>Skill</b>
Calculations with the vertex operators. Calculation based on operator product expansion. Calculations with Virasoro algebra and its representations
<b>Experience</b>
Usage of advanced mathematics to the problems of mathematical physics. Find adequate mathematical tools for concrete problems.

<b>Do you want to specify outcomes in another framework?</b>	Knowledge-Skill-Experience is good enough
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## 9. Assessment Criteria

### Select Assignment 1 Type

Problem Set

### Input Example(s) of Assignment 1 (preferable)

Find the determinant formula for the conformal block for  $c=1$  in resonance case

### Assessment Criteria for Assignment 1

Each task will be evaluated in the same number of points, so the total points for all tasks without asterisks is 50

The solutions of the problems can be discussed with any member of the instructional staff

## 10. Additional Notes