

**Course Title (in English)**

Quantum integrable systems

**Course Title (in Russian)**

Квантовые интегрируемые системы

**Lead Instructor(s)**

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**Status of this Syllabus**

The syllabus is a final draft waiting for form approval

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## 1. Annotation

**Course Description**

The course is devoted to quantum integrable systems. The history of quantum integrable systems starts from 1931 when H.Bethe managed to construct exact eigenfunctions of the Hamiltonian of the Heisenberg spin chain with the help of a special substitution which became famous since that time (ansatz Bethe). In one or another form this method turns out to be applicable to many spin and field-theoretical integrable models. From the mathematical point of view, Bethe's method is connected to representation theory of quantum algebras (q-deformations of universal enveloping algebras and Yangians).

Here is the list of topics which will be discussed in the course.

- Coordinate Bethe ansatz on the example of the Heisenberg model and one-dimensional Bose gas with point-like interaction between particles.
- Bethe ansatz in exactly solvable models of statistical mechanics on the lattice.
- Calculation of physical quantities in integrable models in thermodynamic limit, thermodynamic Bethe ansatz.
- Bethe equations and the Yang-Yang function, calculation of norms of Bethe vectors.
- Quantum inverse scattering method and algebraic Bethe ansatz, quantum R-matrices, transfer matrices, Yang-Baxter equation.
- Functional Bethe ansatz and the method of Baxter's Q-operators, functional relations for transfer matrices, transfer matrices as classical tau-functions.

The knowledge of quantum mechanics and statistical physics for understanding of the course is highly desirable but not absolutely necessary. Out of the physical context ansatz Bethe in its finite-dimensional version is simply a method for diagonalization of big matrices of a special form. In this sense it does not require anything except the basic notions of linear algebra.

#### Course Prerequisites

Students should have basic knowledge of linear algebra, quantum mechanics and statistical physics.

#### 2. Structure and Content

#### Course Academic Level

Master-level

#### Number of ECTS credits

6

| Topic   | Summary of Topic  | Lectures<br>(# of hours) | Seminars<br>(# of hours) | Labs<br>(# of hours) |
|---|---|--------------------------|--------------------------|----------------------|
| Coordinate Bethe ansatz   | Coordinate Bethe ansatz in the spin chain (Heisenberg magnet) and in one-dimensional Bose gas. Bethe wave function. Bethe equations.                      | 6                        | 0                        | 0                    |
| Bethe ansatz in exactly solvable models of statistical mechanics on the lattice | Bethe ansatz in exactly solvable models of statistical mechanics on the lattice. Six-vertex model.  | 6                        | 0                        | 0                    |
| Calculation of physical quantities in integrable models in thermodynamic limit  | Calculation of thermodynamic quantities in integrable models in thermodynamic limit. Thermodynamic Bethe ansatz. Yang's integral equation.                | 6                        |                          |                      |
| Quantum inverse scattering method   | Quantum inverse scattering method and algebraic Bethe ansatz, quantum R-matrices, transfer matrices, Yang-Baxter equation                                 | 4                        | 0                        | 0                    |
| Functional Bethe ansatz   | Functional Bethe ansatz and the method of Baxter's Q-operators, functional relations for transfer matrices, transfer matrices as classical tau-functions. | 4                        | 0                        | 0                    |
| Quantum-classical correspondence  | Functional relations for quantum transfer matrices as the mKP hierarchy of classical integrable equations   | 2                        |                          |                      |

### 3. Assignments

| Assignment Type | Assignment Summary                     |
|-----------------|--|
| Problem Set     | A set of problems on the course.       |
| Homework        | A set of problems for solving at home. |

### 4. Grading

|                           |                      |                           |
|---------------------------|----------------------|---------------------------|
| <b>Type of Assessment</b> | Graded               |                           |
| <b>Grade Structure</b>    | <b>Activity Type</b> | <b>Activity weight, %</b> |
|                           | Homework Assignments | 20                        |
|                           | Class participation  | 20                        |
|                           | Problem Set          | 60                        |

A: Grading Scale

B: 86

C: 76

D: 66

E: 56

F: 46

0

## 5. Basic Information

**Attendance Requirements** Mandatory with Exceptions

| Maximum Number of Students | Maximum Number of Students         |   |
|----------------------------|------------------------------------|---|
|                            | Overall:                           | 7 |
|                            | Per Group (for seminars and labs): | 7 |

**Course Stream** Science, Technology and Engineering (STE)

**Course Term (in context of Academic Year)** Term 3  
Term 4

**Course Delivery Frequency** Every two years

| Students of Which Programs do You Recommend to Consider this Course as an Elective? | Masters Programs                     | PhD Programs |
|---|--------------------------------------|--------------|
|   | Mathematical and Theoretical Physics |              |

**Course Tags** Math  
Physics

## 6. Textbooks and Internet Resources

| Required Textbooks   | ISBN-13 (or ISBN-10) |
|--|----------------------|
| N.M.Bogoliubov, A.G.Izergin, V.E.Korepin, Correlation functions of integrable systems and quantum inverse scattering method, Moscow, Nauka, 1992 | 5-02-014626-7        |

| Recommended Textbooks  | ISBN-13 (or ISBN-10) |
|--|----------------------|
| R.Baxter, Exactly solved models in statistical mechanics, Academic Press, 1982 | 9780486462714        |

| Web-resources (links)   | Description           |
|---|-----------------------|
| <a href="http://people.sissa.it/~ffranchi/BAnotes.pdf">http://people.sissa.it/~ffranchi/BAnotes.pdf</a> | Notes on Bethe ansatz |

## 7. Facilities

## 8. Learning Outcomes

### Knowledge

Students will be familiar with general principles of the theory of quantum integrable systems

### Skill

Students will be able to calculate physical quantities in quantum integrable systems

### Experience

Students will get experience of dealing with quantum integrable systems and exactly solvable models of statistical mechanics

**Do you want to specify outcomes in another framework?**

Knowledge-Skill-Experience is good enough

## 9. Assessment Criteria

**Select Assignment 1 Type**

Problem Set

**Assessment Criteria for Assignment 1**

Correct written solutions of most of the given problems. Ability to explain the solution in conversation.

## 10. Additional Notes

| Web-resources (links)   | Description           |
|---|-----------------------|
| <a href="http://people.sissa.it/~ffranchi/BAnotes.pdf">http://people.sissa.it/~ffranchi/BAnotes.pdf</a> | Notes on Bethe ansatz |