

Course Title (in English) Differential Geometry of Connections

Course Title (in Russian) Дифференциальная геометрия

Lead Instructor(s) Kazarian, Maxim

Status of this Syllabus The syllabus is a work in progress (draft)

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1. Annotation

Course Description

Symplectic geometry is the mathematical language of classical mechanics and hamiltonian dynamic. The course will cover the base of symplectic and contact geometry: symplectic forms, symplectomorphisms, Hamilton vector fields, as well as contact counterparts of these objects. Some applications in differential geometry will include geometry of caustics and wave fronts and their singularities. Among topological applications we will discuss different aspects of Arnold's conjecture on fixed points of symplectomorphisms and its variations.

Course Prerequisites

Differentiable manifolds, diffeomorphisms, vector fields, calculus of differential forms, integration of differential forms and Stokes theorem, Poincare lemma, deRham cohomology. Some knowledge of basic algebraic topology will be useful but not mandatory: fundamental group, homology and Betti numbers, Morse inequalities.

2. Structure and Content

Course Academic Level Master-level course suitable for PhD students

Number of ECTS credits 6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Symplectic structure	Examples of symplectic manifolds. Darboux coordinates. Hamiltonians and Hamiltonian vector fields. Hamiltonian reduction. Symplectomorphisms			
Lagrangian manifolds	Isotropic and Lagrangian submanifolds. Generating functions and generating families. Caustics. Lagrangian Grassmannian. Arnold-Maslov class			
Contact manifolds	Contact structures. Legendrian manifolds. Wave fronts and their singularities.			
Symplectic topological invariants	Arnold's conjecture on the fixed points of symplectomorphisms. Four-cusp theorem. Symplectic incompressibility. Capacities.			

3. Assignments

Assignment Type	Assignment Summary
Problem Set	Problems covering the material of the whole course
Report	Reports on selected topics of the course prepared by its participants (voluntary)

4. Grading

Type of Assessment	Graded	
Grade Structure	Activity Type	Activity weight, %
	Problem Set	50
	Final Exam	50

Grading Scale

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0

5. Basic Information

Attendance Requirements	Mandatory
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Course Term (in context of Academic Year) Term 1
Term 2

Students of Which Programs do You Recommend to Consider this Course as an Elective?	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	

Course Tags Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
Mathematical Methods of Classical Mechanics, V.I.Arnold, 2nd edition	9780387968902

Recommended Textbooks	ISBN-13 (or ISBN-10)
D. McDuff, D. Solomon, Introduction to Symplectic Topology	9780198794905

7. Facilities

8. Learning Outcomes

Knowledge
Symplectic geometry as the modern mathematical language of classical mechanics

Skill
Solving problems on geometry of Lagrangian and Legendrian manifolds, their caustics and wave fronts.

Experience
Experience of working with symplectic manifolds, Hamilton vector fields, generating families for Lagrangian and Legendrian singularities.

Do you want to specify outcomes in another framework? Knowledge-Skill-Experience is good enough

9. Assessment Criteria

Select Assignment 1 Type Problem Set

10. Additional Notes