Course Title (in English)	Statistical mechanics, percolation theory and conformal invariance
Course Title (in Russian)	Статистическая физика, просачивание и конформная инвариантность
Lead Instructor(s)	Shlosman, Semen

Status of this Syllabus	The syllabus is a final draft waiting for form approval
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1. Annotation

Course Description	This is a course on rigorous results in statistical mechanics, random fields and percolation theory. Some of it will be dedicated to the theory of phase transitions, uniqueness or non-uniqueness of the lattice Gibbs fields. We will also study the models at the criticality, where one hopes to find (in dimension 2) the onset of conformal invariance. We will see that it is indeed the case for the percolation and the Ising model. The topics will include: Crossing probabilities Critical percolation The Russo-Seymour-Welsh theory Cardy's formula Parafermionic observables Conformal invariance of two-dimensional percolation Conformal invariance of two-dimensional Ising model O(N)-symmetric models The Mermin–Wagner Theorem The Berezinskii–Kosterlitz–Thouless transition Reflection Positivity and the chessboard estimates Infrared bounds
Course Prerequisites	This course is a continuation of my course on Statistical Physics, so some familiarity with percolation theory and the Ising model is assumed. It is desirable that the students are familiar with the probability theory, measure theory, functional analysis, and complex analysis. Of course, the calculus knowledge is assumed.

2. Structure and Content

Course Academic Level	Master-level
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Crossing probabilities	The behavior of the probabilities of having a left-right crossing in the rectangle box. Subcritical, supercritical and critical case.	1	3	
Critical percolation	The value of the critical probability. Sharpness of the criticality. Power-law decay of connectivities.	1	3	
The Russo-Seymour-Welsh theory	Crossing probabilities for rectangles with arbitrary aspect ratio.	1	3	
Cardy's formula	The exact formula for crossing probabilities.	1	3	
Parafermionic observables		1	3	
Conformal invariance of two-dimensional percolation	The proof of the Theorem of Stanislav Smirnov	1	3	
Conformal invariance of two-dimensional Ising model		1	3	
O(N)-symmetric models	Systems with continuous symmetry in 2D and 3D. Continuous symmetry breaking.	1	3	
The Mermin–Wagner Theorem	No Goldstone bosons in 2D.	1	3	
The Berezinskii–Kosterlitz–Thouless transition	Vortices and the Coulomb gase. Dipoles. Debye screening.	1	3	
Reflection Positivity and the chessboard estimates. Infrared bounds		1	3	

3. Assignments

Assignment Type	Assignment Summary
Test/Quiz	Will be decided later.

4. Grading

Type of Assessment	Pass/Fail
Grade Structure	

Activity Type	Activity weight, %
Final Exam	50
Class participation	30
Report	20

Grading Scale

5. Basic Information

Attendance Requirements	Mandatory with Exceptions

Maximum Number of Students		Maximum Number of Students
	Overall:	20
	Per Group (for seminars and labs):	20

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 3
Course Delivery Frequency	Upon request

Students of Which Programs do You Recommend to Consider this Course as an Elective?	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
Theory of Phase Transitions: Rigorous Results by Ya. G. Sinai	9780080264691
G. Grimmett. Percolation. Springer-Verlag, Berlin, 1999.	978-3-642-08442-3

Recommended Textbooks	ISBN-13 (or ISBN-10)
Statistical Physics L D Landau E.M. Lifshitz	9780080570464

Papers	DOI or URL
Dmitry Chelkak and Stanislav Smirnov. Universality in the 2D Ising model and conformal invariance of fermionic observables. Invent. Math., 189(3):515–580, 2012.	

Web-resources (links)	Description
http://www.unige.ch/math/folks/velenik/smbook/index.html	Statistical Mechanics of Lattice Systems: a Concrete Mathematical Introduction

7. Facilities

Software
Mathematica

Equipment
laptop

Labs for Education	None

8. Learning Outcomes

Knowledge
Theory of random fields, in particular, Markov fields and Gibbs fields. Mathematical theory of phase transitions. Random surfaces. Critical phenomena. Conformal invariance

Skill
Ability to read and understand the literature on rigorous statistical physics and percolation theory, e.g. Journal of Statistical Physics and (some) papers in Communications in Mathematical Physics.
Ability to formulate and sometime also solve problems in the theory of phase transitions and related areas.

Experience
Experience of working in the area of {mathematical physics}cap{probability theory}=ProbaΦ

Do you want to specify outcomes in another framework?	Knowledge-Skill-Experience is good enough

9. Assessment Criteria

Select Assignment 1 Type	Test/Quiz
Input Example(s) of Assignment 1 (preferable)	Prove the existence of phase transition for the Ising model on uniform (infinite) Cayley tree.
Assessment Criteria for Assignment 1	A - for the correct proof. For a wrong or incomplete proof - depending on the sketch.
Select Assignment 2 Type	Test/Quiz
Input Example(s) of Assignment 2 (preferable)	Are there phase transitions of the first order in 2D models with finite range interaction possessing continuous symmetry?
Assessment Criteria for Assignment 2	A - for either a complete proof or for an example. For a wrong or incomplete proof - depending on the sketch.

10. Additional Notes

Web-resources (links)	Description
http://www.unige.ch/math/folks/velenik/smbook/index.html	Statistical Mechanics of Lattice Systems: a Concrete Mathematical Introduction