Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Geometric Representation Theory
Course Title (in Russian)	Геометрическая теория представлений
Lead Instructor(s)	Braverman, Alexander Finkelberg, Michael

Contact Person	Michael Finkelberg
Contact Person's E-mail	fnklberg@gmail.com
Status of this Syllabus	The syllabus is a final draft waiting for form approval

1. Annotation

Course Description	Geometric representation theory applies algebraic geometry to the problems of representation theory. Some of the most famous problems of representation theory were solved on this way during the last 40 years. The list includes the Langlands reciprocity for the general linear groups over the functional fields, the Langlands-Shelstad fundamental Lemma, the proof of the Kazhdan-Lusztig conjectures; the computation of the characters of the finite groups of Lie type. We will study representations of the affine Hecke algebras using the geometry of affine Grassmannians (Satake isomorphism) and Steinberg varieties of triples (Deligne-Langlands conjecture). This is a course for master students knowing the basics of algebraic geometry, sheaf theory, homology and K-theory.
Course Prerequisites	The basic algebraic geometry, sheaf theory, homology and K-theory.

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Affine Grassmannians.	Schubert varieties of finite and infinite codimension, semiinfinite orbits.			
Hyperbolic stalks.	Dimension estimates for the intersection of semiinfinite and G(O)-orbits. Exactness of the hyperbolic stalks.			
Convolution.	Exactness of convolution. Convolution vs. fusion. Commutativity constraint.			
Kazhdan-Lusztig-Ginzburg construction.	Demazure operators in the equivariant K-theory of the Steinberg triple variety. Relation of Borel-Moore homology and Ext-algebra for semismall resolutions.			

3. Assignments

Assignment Type	Assignment Summary
	Problems on the intersection cohomology sheaves on affine Grassmannian.
	Problems on the nearby and vanishing cycles.
	Problems on the Hall algebra and the spherical affine Hecke algebra.

4. Grading

Type of Assessment	Graded
A:	80
B:	70
C:	60

D:	50
E:	40
F:	30

5. Basic Information

Attendance Requirements	Optional

Maximum Number of Students

	Maximum Number of Students
Overall:	10
Per Group (for seminars and labs):	10

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1 Term 2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Course Tags	Math Physics

6. Textbooks and Internet Resources

Required Textbooks	ISBN-10 or ISBN-13
Chriss N., Ginzburg V., Representation theory and complex geometry. Birkhauser, Boston, 2010.	978-0-8176-4937-1

Recommended Textbooks	ISBN-10 or ISBN-13
Macdonald I.G., Symmetric functions and Hall algebras, Clarendon Press, 2015.	978-0-19-873912-8

7. Facilities

8. Learning Outcomes

Knowledge
Geometry of the affine Grassmannians.

Skill
Working knowledge of computations with intersection cohomology sheaves.

Experience
Experience of working with the Hall algebra and spherical affine Hecke algebras.

Do you want to specify outcomes in another framework?	Knowledge-Skill-Experience is good enough

9. Assessment Criteria

Select Assignment 1 Type	Problem Set
Input Example(s) of Assignment 1	1. Prove that the IC sheaves with complex coefficients of the G(O)-orbit closures in the affine Grassmannian of GL(2) are constant. 2. Find the IC stalks with coefficients in the algebraic closure of a finite field of characteristic p of the G(O)-orbit closures in the affine Grassmannian of GL(2). 3. Find the IC stalks of the minimal G(O)-orbit closure in the affine Grassmannian of a simple algebraic group G. 4. Find the hyperbolic stalks in the problem 3 above. 5. Find the usual and hyperbolic stalks of the IC sheaf of the nilpotent cone of a simple Lie algebra g.