Course Title (in English)	Lie groups and Lie algebras, and their representations
Course Title (in Russian)	Группы и алгебры Ли и их представления
Lead Instructor(s)	Olshanski, Grigori

Status of this Syllabus	The syllabus is a final draft waiting for form approval
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1. Annotation

Course Description	We shall begin with the basics of the theory of Lie groups and Lie algebras. Then we shall provide an accessible introduction to the theory of finite-dimensional representations of classical groups on the example of the unitary groups U(N). Tentative plan: linear Lie groups and their Lie algebras; universal enveloping algebras; Haar measure on a linear Lie group; general facts about representations of compact groups and their characters; radial part of Haar measure; Weyl's formula for characters of the unitary groups; Weyl's unitary trick; classification and realization of representations; symmetric functions.
Course Prerequisites	Good knowledge of linear algebra; basics of multivariable calculus; understand the definition of topological space, smooth manifold, tangent space; some knowledge of the basics of representation theory of finite groups (not mandatory, but desirable).

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Lie theory	Definition of Lie group. Linear Lie groups. Subgroups of Lie groups. Exponential map. Definition of Lie algebra. Connections between Lie groups and Lie algebras.			
Lie algebras	Universal enveloping algebra of a Lie algebra. Its center. Symmetric algebra of a Lie algebra. PBW theorem. Representations of sl(2).			
Representations and characters	Haar measure. Weyl's formula for characters. Classification and realization of representations. Introduction to the theory of symmetric functions.			

3. Assignments

Assignment Type	Assignment Summary
Problem Set	About 40 exercises covering the whole material

4. Grading

Type of Assessment	Graded

Grade Structure

Activity Type	Activity weight, %
Class Participation	-45

Grading Scale

A:	80
B:	70
C:	60
D:	50
E:	40
F:	30

5. Basic Information

Attendance Requirements	Optional

Maximum Number of Students

	Maximum Number of Students
Overall:	30
Per Group (for seminars and labs):	

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1 Term 2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-10 or ISBN-13

Recommended Textbooks	ISBN-10 or ISBN-13
William Fulton and Joe Harris, Representation theory (Russian translation available)	978-0-387-97495-8
Jacques Faraut, Analysis on Lie groups. An introduction.	978-0-521-71930-8

Web-resources	Description

7. Facilities

Software

8. Learning Outcomes

Knowledge
Lie Theory. Basics of finite-dimensional representation theory

Skill
Working knowledge of basic constructions in Lie theory and in representation theory of compact groups

Experience
Experience of working with Lie groups, Lie algebras, finite-dimensional representations, and characters

Do you want to specify outcomes in another framework?	Knowledge-Skill-Experience is good enough

9. Assessment Criteria

Input Example(s) of Assignment 1 (preferable)	1. Let M and N be smooth manifolds, $M' \subset M$ and $N' \subset N$ be their closed submanifolds, and $f : M \to N$ be a smooth map such that $f(M') \subseteq N'$. Prove that the restriction of f to M' is a smooth map $M' \to N'$. 2. Show that for smooth manifolds, the properties of being connected space or linearly connected space coincide.
	3. Write explicitly the exponential map for the ax+b group. 4. Find the image of the exponential map for the following Lie groups: a) GL(N,C); b) SO(N); c) the group of real unitriangular $N \times N$ matrices.

10. Additional Notes