Course Title (in English)	Differential topology
Course Title (in Russian)	Дифференциальная топология
Lead Instructor(s)	Gaifullin, Alexander

Status of this Syllabus	The syllabus is a final draft waiting for form approval
Contact Person	Alexander Gaifullin
Contact Person's E-mail	a.gaifullin@skoltech.ru

1. Annotation

Course Description	We plan to discuss two topics, which are central in topology of smooth manifolds, the h-cobordism theorem and theory of characteristic classes. The h-cobordism theorem proved by S. Smale in 1962 is the main (and almost the only) tool for proving that two smooth manifolds (of dimension greater than or equal to 5) are diffeomorphic. In particular, this theorem implies the high-dimensional Poincare conjecture (for manifolds of dimensions 5 and higher). Characteristic classes, in particular, Pontryagin classes are very natural invariants of smooth manifolds. Computation of characteristic classes can help one to distinguish between non-diffeomorphic manifolds. We plan to finish the course with the theorem by J. Milnor on non-trivial smooth structures on the 7-dimensional sphere. This theorem is based both on methods of Morse theory and theory of characteristic classes
Course Prerequisites	Differential topology, algebraic topology, Morse theory, theory of characteristic classes

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Homology and cohomology	De Rham cohomology. Singular homology. Pairing between homology and cohomology. Multiplication in cohomology and intersection of cycles. Poincare duality.	3	20	0
h-cobordism theorem	Morse functions. Cobordisms corresponding to critical points. Morse inequalities. Lefschetz theorem on hyperplane sections. Smale's h-cobordism theorem. High-dimensional Poincare conjecture.	4	25	0
Characteristic classes	Principal bundles and their characteristic classes. Chern-Weil theory. Chern classes and Pontryagin classes. Integral Chern classes and Pontryagin classes. Smooth structures on the 7-dimensional sphere.	4	25	0

3. Assignments

Assignment Type	Assignment Summary
Homework	List of problems on homology and cohomology theories, Morse theory, and theory of characteristic classes

4. Grading

Type of Assessment	Graded

Grade Structure

Activity Type	Activity weight, %
Final Exam	100

Grading Scale

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0

5. Basic Information

Attendance Requirements	Optional

Maximum Number of Students		
		Maximum Number of Students
	Overall:	20
	Per Group (for seminars and labs):	20

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 3 Term 4
Course Delivery Frequency	Every year

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Please List the Teaching Assistants (TAs) You Propose for Your Course

First Name	Last Name
Denis	Gorodkov

Course Tags	Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-10 or ISBN-13
B.A. Dubrovin, A.T. Fomenko, S.P. Novikov, Modern Geometry — Methods and Applications.	9780387976631
J. Milnor, Morse theory	9780691080086

Recommended Textbooks	ISBN-10 or ISBN-13
J. Milnor, Lectures on the h-cobordism theorem.	9780691624556
R. Bott, L.W. Tu, Differential forms in algebraic topology.	978-1-4757-3951-0

7. Facilities

8. Learning Outcomes

Knowledge
Basic knowledge on homology and cohomology, Morse theory, differential geometric approach to characteristic classes.

Skill
Skills of computation of homology and cohomology groups and rings and characteristic classes of spaces and manifolds.

Experience
Experience of studying hard mathematical theory in the field of geometry and topology.

Do you want to specify outcomes in another framework?	Knowledge-Skill-Experience is good enough

9. Assessment Criteria

Select Assignment 1 Type	Homework
Input Example(s) of Assignment 1 (preferable)	Find critical points and their indices for the function on CP(n) given by $c_0|z_0|^2+\ldots+c_n|z_n|^2$ for the given real constants $c_1,\ldots,c_n$.
Assessment Criteria for Assignment 1	The critical points and their indices must be found correctly. The absence of other critical points must be proved.

10. Additional Notes