

Course Title (in English)	Elliptic operators in topology of manifolds
Course Title (in Russian)	Эллиптические операторы в топологии многообразий
Lead Instructor(s)	Gaifullin, Alexander
Is this syllabus complete, or do you plan to edit it again before sending it to the Education Office?	The syllabus is a final draft waiting for approval (once approved the syllabus will be published on the public web-site and other systems)
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## 1. Annotation

### Course Description

The course will be devoted to applications of elliptic differential operators in topology of manifolds. We start with basics of Hodge theory including a detailed proof of the existence of the harmonic representative in a de Rham cohomology class. This proof is based on the usage of Sobolev spaces, and we will pay attention to this technique. Further, we proceed with the notion of the index of an elliptic differential operator towards the Atiyah-Singer theorem and its applications. Examples of several most important operators will be studied in details.

### Course Prerequisites / Recommendations

Students should be familiar with the standard course of differential geometry, including differential forms and de Rham cohomology. Besides, students should be familiar with theory of vector bundles and connections in them. It is desirable that students know basic facts of functional analysis and partial differential equations.

## 2. Structure and Content

Course Academic Level

Master-level course suitable for PhD students

Number of ECTS credits

6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Hodge theory	Hodge theorem: Existence of a harmonic representative in a de Rham cohomology class. Sobolev spaces.	5	18	
Atiyah-Singer theorem	Elliptic differential operators in vector bundles. Examples. Index of an elliptic operators. Elliptic complexes. Atiyah-Singer theorem. Its applications.	6	20	

### 3. Assignments

### 4. Grading

Type of Assessment

Graded

Grade Structure

Activity Type	Activity weight, %
Final Exam	100

### Grading Scale

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Attendance Requirements Mandatory with Exceptions

## 5. Basic Information

Course Term (in context of Academic Year) Term 1-2

Course Delivery Frequency Every year

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Course Tags Math

## 6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
Wells, RO, Differential analysis on complex manifolds, 1973	
Palais, RS, Seminar on the Atiyah-Singer index theorem, 1965	

Recommended Textbooks	ISBN-13 (or ISBN-10)
Griffits, P, Harris, J, Principles of algebraic geometry	
Л. Шварц, Комплексные аналитические многообразия. Эллиптические уравнения с частными производными, М.: Мир, 1982	

## 7. Facilities

## 8. Learning Outcomes

### Knowledge

Hodge theory. Atiyah-Singer theorem.

### Skill

Usage of elliptic complexes in topology of manifolds.

### Experience

Experience of applying analytic methods (Sobolev spaces, differential equations, etc.) in topology.

## 9. Assessment Criteria

Input or Upload Example(s) of Assignment 1:

Input or Upload Example(s) of Assignment 2:

Input or Upload Example(s) of Assignment 3:

Input or Upload Example(s) of Assignment 4:

Input or Upload Example(s) of Assignment 5:

## 10. Additional Notes