Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Modern Dynamical Systems
Course Title (in Russian)	Современные динамические системы
Lead Instructor(s)	Alexandra Skripchenko
Is this syllabus complete, or do you plan to edit it again before sending it to the Education Office?	The syllabus is a final draft waiting for form approval
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1. Annotation

Course Description

Dynamical systems in our course will be presented mainly not as an independent branch of mathematics but as a very powerful tool that can be applied in geometry, topology, probability, analysis, number theory and physics. We consciously decided to sacrifice some classical chapters of ergodic theory and to introduce the most important dynamical notions and ideas in the geometric and topological context already intuitively familiar to our audience. As a compensation, we will show applications of dynamics to important problems in other mathematical disciplines. We hope to arrive at the end of the course to the most recent advances in dynamics and geometry and to present (at least informally) some of results of A. Avila, A. Eskin, M. Kontsevich, M. Mirzakhani, G. Margulis.
In accordance with this strategy, the course comprises several blocks closely related to each other. The first three of them (including very short introduction) are mainly mandatory. The decision, which of the topics listed below these three blocks would depend on the background and interests of the audience.

Course Prerequisites / Recommendations	We expect our audience to be familiar with basic differential geometry, basic topology and basic measure theory.

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Introduction	We will introduce dynamical systems using the most elementary examples — rotation of the circle and continued fractions.	1	2	
Dynamics and geometry	In this part we will check how dynamical methods can be used to study one of the most classical notions of differ- ential geometry — geodesics on surfaces of negative curvature. Here is our approximate plan: (1) Introduction to hyperbolic geometry. Möbius transformations. Fuchsian groups. (2)The eight geometries of the geometrization conjecture by W. Thurston. (3) Geodesics on surfaces of negative curvature. The geodesic flow and its properties. (4) Geodesic flow on modular curve as a continued fraction map. (5) Teichmüller space. Teichmüller geodesic flow. (6)Counting of simple closed geodesics: results by M. Mirzhakhani.	4	16	
Dynamics and topology	In this part we stay again on a Riemann surface but now we would like to have an almost flat metrics on it and to consider related geodesic flow (equivalently, we study measured foliations on such a surface). The purpose of this block is to make a crash course in ergodic theory with a topological interpretation of the main notions and results. The plan is as follows: (1) Interval exchange transformations (IET) as natural generalizations of continued fractions. (2) IET as the first return maps on transversal for measured foliations on oriented surface. Poincaré recurrence theorem. (3) Key ergodic properties: minimality, ergodicity, number of invariant measures (illustrated by IET). (4) Multiplicative ergodic theorem. Topological interpretation of Lya- punov exponents. Sums of Lyapunov exponents as uniform bounds for degrees of holomorphic subbundles. (5) Anosov and Pseudoanosov diffeomorphisms of surfaces. Introduction to hyperbolic dynamics (Markov partitions, invariant measures etc). (6)∗ IET and billiards in rational polygons. Counting billiard orbits of special types. Complexity of orbit. (7)∗ Action of GL(2,R) on the moduli space of Abelian differentials. Ehrenfest wind-tree model of Boltzmann gas.	4	16	
Dynamics and number theory	This block (that can be chosen by our auditory) is dedicated to homogeneous dynamics and its applications to famous conjectures in number theory, such as Oppenheim conjecture (solved) and Littlewood conjecture (still open). We mainly will follow G. Margulis work in this direction.	2	8	

Dynamics and analysis.	This block can be more interesting for fu- ture specialists in mathematical physics and numerical analysis. We plan to discuss the notion of transfer operator and its spectral gap, Perron-Frobenius theorem and its generalization by D. Ruelle, zeta-function and its interpre- tation in terms of transfer operator.	1	4	
Dynamics and probability.	This block is dedicated to thermodynamical formalism and its application for dynamical systems. We discuss beautiful ideas by R. Dobrushin, on the one hand, and R. Bowen — on the other, and show how to find Gibbs measures for dynamical systems that can be presented as a symbolic shift.	1	2	

3. Assignments

Assignment Type	Assignment Summary
Problem Set	The students will be given a list of problems that cover the course and are supposed to show their understanding of basic notions and ability to use main technical tools.
Project	The students will be given some research papers from the field (both modern and classical) and will try to present it to their colleagues. In order to get the highest mark students are supposed to show their deep understanding of the subject of the given papers as well as ability to explain it to other people in the audience. We will also take into consideration the level of independence in the choice of the subject as well as quality and amount of the used sources.

4. Grading

Type of Assessment Graded

Grade Structure

Activity Type	Activity weight, %
Projects	60
Final Exam	40

Grading Scale

A:	86
B:	76
C:	66
D:	56
E:	46

F:	0
Attendance Requirements	Optional with Exceptions

5. Basic Information

Maximum Number of Students

	Maximum Number of Students
Overall:	50
Per Group (for seminars and labs):	30

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 3 Term 4
Course Delivery Frequency	Every year

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Course Tags	Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
S. Katok, Fuchsian groups, University of Chicago Press, Chicago and London, 1992 (Russian translation: Faktorial Press, Moscow, 2002)	9780226425825
Ya. Sinai, Introduction to ergodic theory Princeton University Press, 1977	9780691081823

Recommended Textbooks	ISBN-13 (or ISBN-10)
W. Thurston, Geometry and topology of three-manifolds, Princeton University Press, 1997 (Russian translation: MCCME, 2001)	9780691083049
F. Dal'bo, Geodesic and Horocyclic trajectories, Springer Urtext (2011).	9780857290731

Web-resources (links)	Description
http://www.college-de-france.fr/media/jean-christophe-yoccoz/UPL15305_PisaLecturesJCY2007.pdf	lecture notes by Fields medalist Jean-Christophe Yoccoz on interval exchange transformations and related aspects of ergodic theory
http://jointmathematicsmeetings.org/meetings/national/jmm/margulis_colloq_lect_08.pdf	Lecture notes by Grigory Margulis on Number theory and homogeneous dynamics
http://www.weizmann.ac.il/math/sarigo/sites/math.sarigo/files/uploads/tdfnotes.pdf	Lecture notes by Omri Sarig on thermodynamical formalism for countable Markov shift
http://www.weizmann.ac.il/math/sarigo/sites/math.sarigo/files/uploads/transferoperatorcourse.pdf	Lecture notes by Omri Sarig on transfer operators

7. Facilities

8. Learning Outcomes

Knowledge
Students will be introduced to some classical results as well as the most modern achievements in dynamical systems and will learn how to apply these results in other branches of mathematics, for example, in geometry, topology, probability and number theory.

Skill
Students are supposed to become familiar with the most advanced techniques that are applied in dynamical systems.

Experience
Students will read and understand deeply plenty of celebrated papers in dynamics. Some of the materials we are supposed to study were mentioned by the Fields committee as a main motivation to award the Fields medal in the last few decades.

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Select Assignment 1 Type	Problem Set
Or Upload Example(s) of Assignment 1	https://ucarecdn.com/d1aa2959-2d47-46b9-9ca0-0571d712e7df/
Assessment Criteria for Assignment 1	In order to get the full score for a given problem the student should calculate correctly both the genus of the surface and the number of zeros of the form. If the student knows the idea but is not able to provide the precise calculation, half of scores can be given. If no correct approach to the problem was suggested, 0 score is given.

Input or Upload Example(s) of Assigment 2:

Select Assignment 2 Type	Project
Or Upload Example(s) of Assignment 2	https://ucarecdn.com/82a7a1b7-c9f1-457f-95ff-c5e15be68d7b/
Assessment Criteria for Assignment 2	The student should present a summary of the paper at our problem solving class in front of other students and professors. If the student shows reasonable level of understanding of the problem, it is enough to complete the assignment.

Input or Upload Example(s) of Assigment 3:

Input or Upload Example(s) of Assigment 4:

Input or Upload Example(s) of Assigment 5:

10. Additional Notes

Free Style Comments (if any)	This year I plan to teach this course with some contribution of my colleagues - Sergey Lando and Anton Zorich. Their names should be also added to the syllabus somehow (even if most part of lectures will be delivered by myself).