

<b>Course Title (in English)</b>	Representations of classical groups and related topics
<b>Course Title (in Russian)</b>	Представления классических групп и связанные с ними вопросы
<b>Lead Instructor(s)</b>	Olshanski, Grigori
<b>Is this syllabus complete, or do you plan to edit it again before sending it to the Education Office?</b>	The syllabus is a final draft waiting for approval (once approved the syllabus will be published on the public web-site and other systems)
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## 1. Annotation

### Course Description

The course is focused on fundamental results of the representation theory of classical matrix groups, which find numerous applications in various domains of mathematics. Particular attention will be paid to links with algebraic combinatorics.

Tentative program:

- Characters of classical groups (general linear, orthogonal, and symplectic).
- Second Weyl character formulas
- Classical invariant theory and applications
- Representations in traceless tensors
- Brauer duality
- Highest weight representations and Littlewood formulas
- Center of universal enveloping algebra
- Perelomov-Popov theorem
- Capelli identity
- Multidimensional interpolation polynomials
- Binomial formula for characters
- Okounkov's quantum immanants
- Applications to asymptotic representation theory

**Course Prerequisites / Recommendations**

Good knowledge of basic algebra (groups, rings, modules). Good knowledge of linear algebra. Basics of Lie groups/Lie algebra theory. Some acquaintance with basics of general representation theory of finite/compact groups would be helpful. Basics of functional analysis (Hilbert space, bounded operators on Hilbert space).

## 2. Structure and Content

**Course Academic Level**

Master-level course suitable for PhD students

**Number of ECTS credits**

3

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Finite-dimensional representations and characters	<ul style="list-style-type: none"> <li>- Characters of classical groups (general linear, orthogonal, and symplectic).</li> <li>- Second Weyl character formulas</li> <li>- Classical invariant theory and applications</li> <li>- Representations in traceless tensors</li> <li>- Brauer duality</li> <li>- Highest weight representations and Littlewood formulas</li> </ul>	27	-	-
Universal enveloping algebra and its center	<ul style="list-style-type: none"> <li>- Highest weight representations</li> <li>- Center of universal enveloping algebra</li> <li>- Perelomov-Popov theorem</li> <li>- Capelli identity</li> </ul>	27		
Multidimensional interpolation polynomials and applications	<ul style="list-style-type: none"> <li>- Multidimensional interpolation polynomials</li> <li>- Binomial formula for characters</li> <li>- Okounkov's quantum immanants</li> <li>- Applications to asymptotic representation theory</li> </ul>	27		

## 3. Assignments

Assignment Type	Assignment Summary
Problem Set	exercises covering the whole material

## 4. Grading

Type of Assessment Graded

Grade Structure

Activity Type	Activity weight, %
Homework Assignments	-2

## Grading Scale

A: 80

B: 70

C: 60

D: 50

E: 40

F: 0

Attendance Requirements Optional

## 5. Basic Information

Maximum Number of Students

	Maximum Number of Students
Overall:	30
Per Group (for seminars and labs):	

Course Stream Science, Technology and Engineering (STE)

Course Term (in context of Academic Year) Term 1  
Term 2

Course Delivery Frequency n/a

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags

Math

## 6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
n/a	

Recommended Textbooks	ISBN-13 (or ISBN-10)
Hermann Weyl. The classical groups. Their invariants and representations (Russian translation available)	978-0691057569
D.P.Zhelobenko. Compact Lie groups and their representations (Translation of Russian edition)	978-0-8218-1590-8

Papers	DOI or URL
A.Okounkov and G.Olshanski, Shifted Schur functions	<a href="https://arxiv.org/abs/q-alg/9605042">https://arxiv.org/abs/q-alg/9605042</a>
A.Okounkov and G.Olshanski, Shifted Schur functions II. Binomial formula for characters of classical groups and applications	<a href="https://arxiv.org/abs/q-alg/9612025">https://arxiv.org/abs/q-alg/9612025</a>

## 7. Facilities

## 8. Learning Outcomes

Knowledge
Fundamental results about finite-dimensional representations of classical groups, their characters, and related topics.

### Skill

Working knowledge of fundamental results and constructions in combinatorial representation theory.

### Experience

Experience of working with characters of classical groups.

## 9. Assessment Criteria

Input or Upload Example(s) of Assignment 1:

Select Assignment 1 Type

Homework Assignments

Input Example(s) of Assignment 1 (preferable)

1. Compute the dimension of an irreducible representation with a given highest weight.
2. Decompose a given representation into irreducibles.
3. Deduce the Jacobi-Trudi formula from the Cauchy identity

Assessment Criteria for Assignment 1

The problems will vary in difficulty. Each solved problem may give 1, 2 or 3 points. The total score is calculated according to the formula  $\min(100, 200 \cdot S/N)$ , where  $S$  denotes the total number of points obtained and  $N$  denotes the maximal possible number of points.

Input or Upload Example(s) of Assignment 2:

Input or Upload Example(s) of Assignment 3:

Input or Upload Example(s) of Assignment 4:

Input or Upload Example(s) of Assignment 5:

## 10. Additional Notes