Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Critical points of functions
Course Title (in Russian)	Критические точки функций
Lead Instructor(s)	Kazarian, Maxim
Is this syllabus complete, or do you plan to edit it again before sending it to the Education Office?	The syllabus is a final draft waiting for approval (once approved the syllabus will be published on the public web-site and other systems)
Contact Person	Maxim Kazarian
Contact Person's E-mail	mkazarian@hse.ru

1. Annotation

Course Description

The theory of critical points of functions is of the main subjects of Singularity theory studying local geometry of singularities of differentiable maps as well as its relationship with global topological invariants of manifolds.
In the course we will discuss classification of critical points, its relationship with the ADE-series of simple Lie algebras and the corresponding reflection groups, their deformations and adjacencies. The study of a local topological structure of singularities will include description of Milnor fiber and vanishing cycles. We will discuss also application of the theory critical points to the study of caustics and wave fronts in geometric optics and classical mechanics, as well as enumeration of contact singularities in complex projective geometry.

Course Prerequisites / Recommendations	Basic courses of calculus and linear algebra. Familiarity with basic notions of topology is approved

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Functions in one variables	Classification of critical functions in one variables. Normal forms. Deformations. Germs and jets.	1	4	
Morse lemma	Nondegenerate and degenerate critical points. Second differential. Morse Lemma. Homotopic method. Parametric Morse lemma. Corank	1	4	
Classification of simple singularities	Right and Left-Right equivalence. Stable equivalence. Newton diagram and Newton ruler. Examples of non-simple singularities. Classification of simple singularities.	1	4	
Milnor number	Gradient ideal. Local algebra. Milnor number. Milnor number of aquasihomogeneous singularity	1	4	
Versal deformations	Versal and infinitesimaly versal singularities. Versality theorem	1	4	
Milnor fiber	Topology of Milnor fiber. Vanishing cycles. Intersection form. Dynkin diagram. Stabilization	1	4	
Monodromy	Monodromy operator and monodromy group. Lefschetz formula.	2	8	
Singularities of caustics and wave fronts	Symplectic and contact manifolds. Lagrangian and Legendrian submanifolds. Caustics and wave fronts. Generating families of functions and their discriminants.	1	4	
Thom polynomials	Chern classes of vector bundles. Lagrangean and Legendrean characteristic classes. Thom polynomials for critical point singularities.	1	4	
Enumeration of contact singularities	Applications of Thom polynomials to enumeration of contact singularities. Enumerative invariants of curves and surfaces in projective complex spaces	1	4	

3. Assignments

Assignment Type	Assignment Summary
Problem Set	Various kinds of problems on the subject of the course, both of computational type and proofs. Partially discussed during seminars and partially left to solve at home.

4. Grading

Type of Assessment	Graded

Grade Structure

Activity Type	Activity weight, %
Problem Set	25
Midterm Exam	25
Final Exam	50

Grading Scale

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0

5. Basic Information

Maximum Number of Students

	Maximum Number of Students
Overall:	15
Per Group (for seminars and labs):	

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1 Term 2
Course Delivery Frequency	on request

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
V.I. Arnold, A.N. Varchenko, S.M. Gusein-Zade, Singularities of Differentiable Maps	9781461295891

Recommended Textbooks	ISBN-13 (or ISBN-10)
V.I. Arnold; V.V. Goryunov; O.V. Lyashko, V.A.Vassiliev, Singularity Theory	9783540637110

7. Facilities

8. Learning Outcomes

Knowledge
Basic invariants of critical points: corank, Milnor number, modality, local algebra, normal forms
ADE-classification of simple singularities. Its relationship with reflection groups and simple Lie algebras
Milnor fiber and vatishing topology of singuarities. Monodromy and Pickard-Lefschetz formula.
Characteristic classes associated with critical point function singularities and enumeration of contact singularities in complex projective geometry.

Skill
Ability to manipulating with formal power series, applying changes of variables, applying Newton diagrams and the method of Newton ruler.
Applying homotopy method to reducing to normal forms
Developing topological intuition in the work with Milnor fiber and vanishing cycles.
Ability to apply the theory of Thom polynomials to practical computation of enumerative invariants in projective algebraic geometry.

Experience
Independent solving problems in the domain of critical points of functions
Formulating and proving theorems of in the area of critical point function singularities

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Select Assignment 1 Type	Problem Set
Or Upload Example(s) of Assignment 1	https://ucarecdn.com/e161cc6c-d3ad-4729-9c10-1391ab75c47d/
Assessment Criteria for Assignment 1	out of 10 points each problem, depending on completeness of solution

Input or Upload Example(s) of Assigment 2:

Input or Upload Example(s) of Assigment 3:

Input or Upload Example(s) of Assigment 4:

Input or Upload Example(s) of Assigment 5:

10. Additional Notes