HW1

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Readable solutions of the problems will be sent as photo or scan or pdf-file 16.02 to alevin57@gmail.com before 17.00

The Eisenstein trigonometric functions are the following series: $\varepsilon_n(x) = \sum_{-\infty}^{\infty}(x+\mu)^{-n}$ for $n > 1$; $\varepsilon_1(x) = \lim_{M\to\infty}\sum_{\mu=-M}^{M}(x+\mu)^{-1}$

1) Prove a) for $n > 1$ the function $\varepsilon_n(x)$ is periodic: $\varepsilon_n(x+1) = \varepsilon_n(x)$

b) $\varepsilon_n(-x) = (-1)^n\varepsilon_n(x)$

c)$\varepsilon_n(x)' = -n\varepsilon_{n+1}(x)$

For this prove absolute uniform convergence of series on the bounded sets and apply standard theorems from Calculus (with precise formulations).

2.Put $\gamma_m = \sum_{\mu\neq 0}\mu^{-m}$. Prove that this series absolutely converges for $m \geq 2$

3.a)For $|x| < 1$ $\varepsilon_1(x) = x^{-1} - \sum_{m=2}^{\infty}\gamma_m x^{m-1}$.

b) deduce such a expansion for ε_m

4.Deduce from

$$\frac{1}{(x-\mu)^2}\frac{1}{(y-\nu+\mu)^2} =$$

$$= \frac{1}{(x+y-\nu)^2}\left(\frac{1}{(x-\mu)^2} + \frac{1}{(y-\nu+\mu)^2}\right) +$$

$$+\frac{2}{(x+y-\nu)^3}\left(\frac{1}{(x-\mu)} + \frac{1}{(y-\nu+\mu)}\right).$$

the identity:

$$\varepsilon_2(x)\varepsilon_2(y) = \varepsilon_2(x+y)\left(\varepsilon_2(x)+\varepsilon_2(y)\right) + 2\varepsilon_3(x+y)\left(\varepsilon_1(x)+\varepsilon_1(y)\right)$$