Лекция 9. 19.03.2021.

μ_i не обязательно разные

$\mu \vdash d \Rightarrow \mu_1 + \dots + \mu_m = d$ (кол-во циклов)

перестановки $\in S_d$

Перечисление Детских рисунков. Окончание.

Рекурсия для $N_{k,\ell}(\mu)$: $= N_{k,\ell}(\mu_1 \dots \mu_m)$

$$\mu_1 N_{k,\ell}(\mu_1 \dots \mu_m) = \sum_{j=2}^{m} (\mu_1 + \mu_j - 1) N_{k,\ell}(\mu_1 + \mu_j - 1, \mu_2, \dots, \hat{\mu}_j, \dots \mu_m) + (\mu_1 - 1)\big(N_{k-1,\ell}(\mu_1 - 1, \mu_2 \dots \mu_m) + N_{k,\ell-1}(\mu_1 - 1, \mu_2, \dots, \mu_m)\big) +$$

$K + \ell + m - c = 2 - 2g$

$$+ \sum_{i+j=\mu_1-1} ij \Big(N_{k,\ell}(i, j, \mu_2 \dots \mu_m) + \sum_{\substack{k_1+k_2=k \\ \ell_1+\ell_2=\ell}} \sum_{I \sqcup J = \{2 \dots m\}} N_{k_1,\ell_1}(i, \mu_I) N_{k_2,\ell_2}(j, \mu_J) \Big)$$

$\mu_3, \mu_4 \dots \mu_m$

$$F(s, u, v; p_1, p_2, \dots) = \sum_{k,\ell,m} \frac{1}{m!} \sum_{\mu \vdash d} N_{k,\ell}(\mu)\, s^d u^k v^\ell p_{\mu_1} p_{\mu_2} p_{\mu_3} \dots p_{\mu_m}$$

$$\frac{\partial F}{\partial p_{\mu_1}} = \dots + t_{\mu_1} N_{k,\ell}(\mu_1 \dots \mu_m) p_{\mu_2} p_{\mu_3} \dots p_{\mu_m} + \dots$$

$$\frac{n+1}{s} \frac{\partial F}{\partial p_{n+1}} = \sum_{j=1}^{\infty} p_j (n+j) \frac{\partial F}{\partial p_{n+j}} + (u+v) n \frac{\partial F}{\partial p_n} + \sum_{\substack{i+j=n \\ i,j \geq 1}} ij \left(\frac{\partial^2 F}{\partial p_i \partial p_j} + \frac{\partial F}{\partial p_i} \frac{\partial F}{\partial p_j} \right) + \delta_{0,n} uv$$

если $n=0$ то прибавить uv

$n+1 = \mu_1$

$\mu_j = j$?

$\mu_1 + \mu_j - 1 = n + \mu_j$

$$\sum a_i x^i \cdot \sum b_j x^j \quad [x^k] = \sum_{i+j=k}$$