$$\mathrm{Tr}(dH)^2 = \mathrm{Tr}\Big((d\Lambda)^2 + \underbrace{\Lambda\, dU^{\dagger} \Lambda\, dU} - \Lambda\, dU\, dU\, \Lambda - dU \Lambda \Lambda\, dU + dU \Lambda\, dU\, \Lambda \Big)$$

$$\begin{pmatrix} \lambda_{kk} & 0 \\ 0 & \lambda_{kk} \end{pmatrix}$$

$$\sum_{k=1}^{N} (d\lambda_{kk})^2 + \;=\; 2\,\mathrm{Tr}\big(\Lambda^2 (dU)^2\big)$$

$$+\,2\,\Lambda\, dU\, \Lambda\, dU \qquad \sum_{ij} \lambda_{ii}\, dU_{ij}\, \lambda_{jj}\, dU_{ji}$$

$$-2\sum_{1\le k,j\le N} \lambda_{kk}^2 \boxed{du_{kj}\, du_{jk}} + 2\sum_{1\le k,j\le N} \lambda_{kk}\lambda_{jj}\, du_{kj}\, du_{jk}$$

$$\overline{\quad jk \quad}$$

$$du_{kj}\cdot du_{jk} = (dx_{kj})^2 + (dy_{kj})^2$$

$$-2\lambda_{kk}^2\, du_{kk}\, du_{kk}$$

$$2\left((\lambda_{kk})^2 + (\lambda_{jj})^2 - 2\lambda_{kk}\lambda_{jj}\right) \qquad \left((dx_{kj})^2 + (dy_{kj})^2\right)^2$$

$$2\,(\lambda_{kk}-\lambda_{jj})^2\left((dx_{kj})^2+(dy_{kj})^2\right) \qquad N^2-N$$

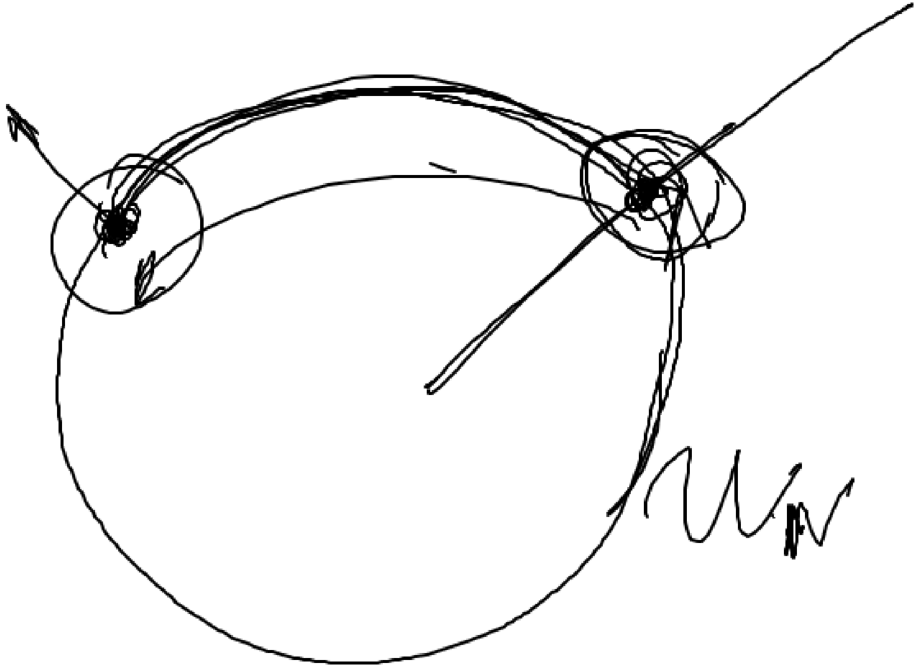
$$\prod_{1 \le i < j \le N} (\lambda_{ii} - \lambda_{jj})^2 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ \lambda_{11} & \lambda_{22} & & & \lambda_{NN} \\ & & & & \\ \lambda_{11}^{N-1} & & & & \lambda_{NN}^{N-1} \end{vmatrix}^2$$

$N^2 \times N^2$

$$\frac{T_n(N)}{(2n-1)!!}$$

$$\frac{T_n(N)}{2^N} = \frac{1}{\pi^{N^2}} \int_{\mathcal{H}} \underbrace{\mathrm{Tr}\left((H)^{2n}\right)}_{U^{+} H U} e^{-\mathrm{Tr} H^2} \sqrt{G}\, d \ldots\ldots$$

$$= \frac{1}{\pi^{N}} \int_{\Lambda \times U} \mathrm{Tr}\left(\Lambda^{2n}\right) e^{-\mathrm{Tr}\Lambda^2} \prod_{i<j} (\lambda_{ii} - \lambda_{jj})^2 \, dh_{11} \ldots dh_{NN}\, dU$$



$U^\dagger M U$

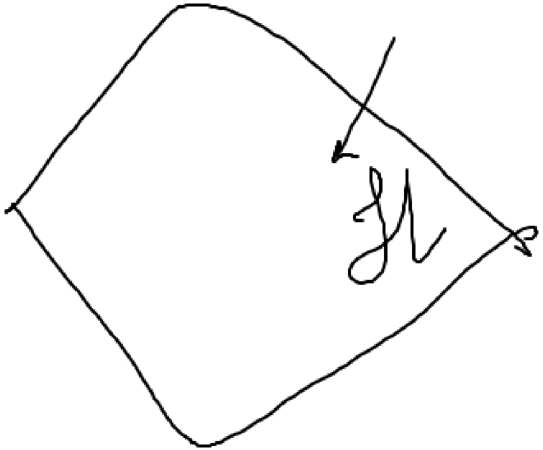
$\mathrm{Tr}(dM\, dM^\dagger)$

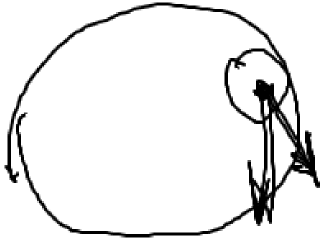
$$M_{N\times N} \supset \mathcal{U}_N$$

$$\mathrm{Tr}(dM\,dM^\dagger)$$

$$\mathbb{R}^2$$

$$\boxed{dx^2+dy^2}$$





$$\frac{1}{\pi^{N^2} V_{U_N}} \int_{\mathbb{R}^N} \prod \lambda_{11}^{2n} \, \underline{\underline{e^{-\sum \lambda_{kk}^2}}} \underbrace{\prod_{k<j} (\lambda_{kk}-\lambda_{jj})^2 \, d\lambda_{11} \dots d\lambda_{NN}}_{N}$$

$$= \frac{1}{\sqrt{\pi}^{N} V_{U_N}} \int_{\mathbb{R}} \underbrace{\tilde{P}_{2N-2+2n}}_{2N-2}(\lambda_{11}) \, e^{-\lambda_1^2} \, d\lambda_1 = \frac{\quad}{2^{N-1+n}} \int \lambda_{11}^{2N-2+2n} e^{-\lambda_1^2} d\lambda_1$$

$$2^n \frac{(2n+2k-1)!!}{2^{k+n}\,(2n-1)!!} \qquad \cdot \qquad \frac{2N-2+2n}{2^{2n+2k}}$$

$$= \frac{\overbrace{(2n+2k-1)\,(\ldots)\,(2n+1)}^{k}}{2^k}$$

$$\frac{I_n(N)}{(2n-1)!!} = \sum \xi \, \tilde{I}_n(k)$$