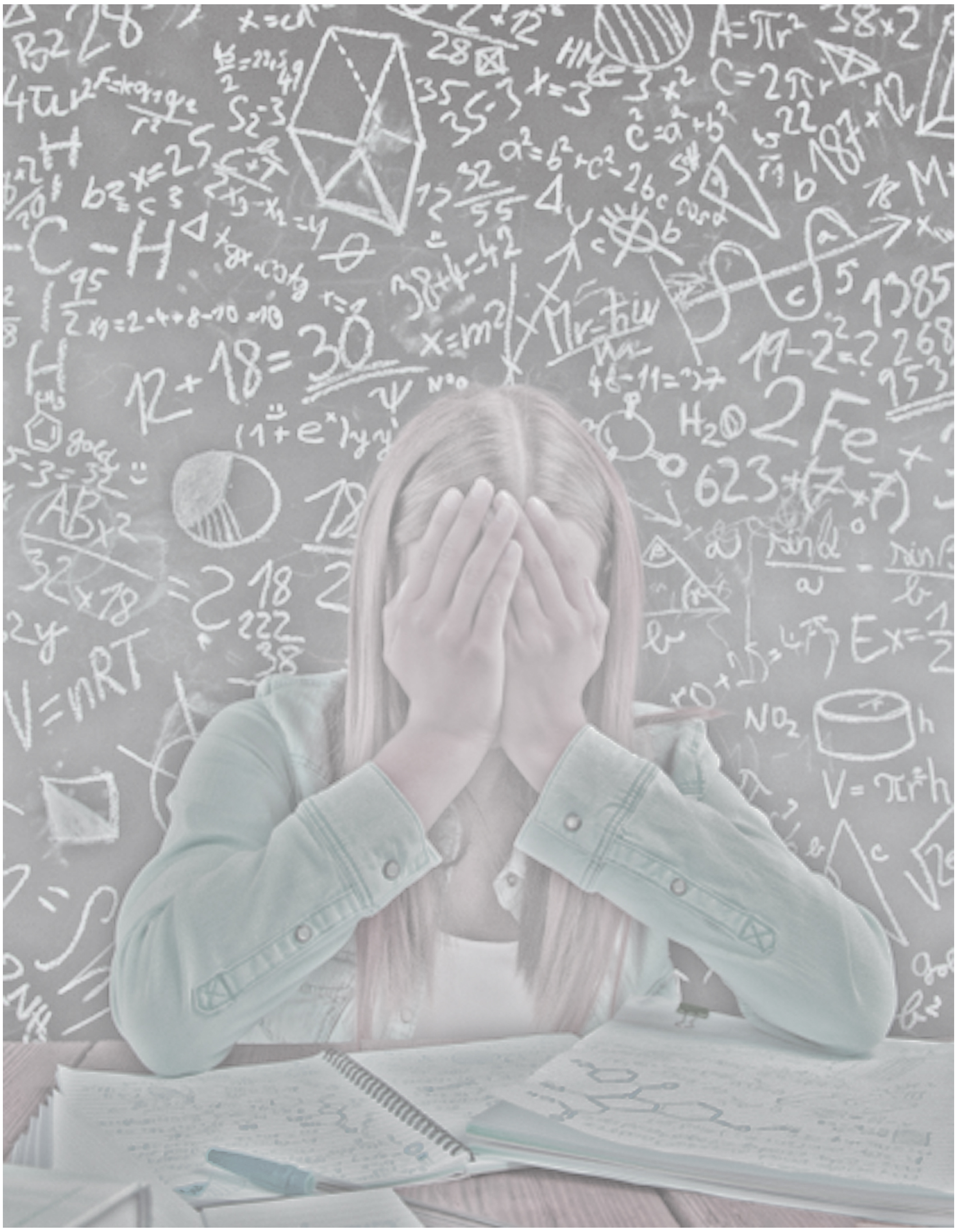
NATIONAL RESEARCH UNIVERSITY «HIGHER SCHOOL OF ECONOMICS»

FACULTY OF MATHEMATICS

# Courses and Seminars in English Offered by the Faculty of Mathematics in 2018/19 Academic Year



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## Courses and seminars

Everywhere below, typesetted in **bold letters** are so called «heavy» courses and seminars holding 2 classes per week and worthing 5 credits per semester. All the other, «slim» courses and seminars hold 1 class per week and worth 3 credits per semester. The subdivision of classes into «courses» and «seminars» is caused often by some bureaucratic reasons and may not reflect an actual style of classes. More detailed information about every course or seminar can be found at the special page describing this course or seminar, see the Table of Contents on p. 2,3.

### Primary Level Courses

These courses are recommend for filling gaps in the previous mathematical education and making the first steps towards the chosen specialization. In the Table of Contents on p. 2,3, the references to these courses are typesetted in *italic*. The course «Mathematical methods of science» is obligitory for the first year students of the MSc program «Mathematics».

FALL

- **Algebra and Arithmetics, V. S. Zhgoon**
- **Classical Analysis and ODE, T. Takebe**
- **Introduction to Combinatorial Theory, Yu. M. Burman**
- **Topics in differential and algebraic topology, P. E. Pushkar**
- **Algebraic Geometry: A First Geometric Look, V. S. Zhgoon**
- **Galois Theory, C. Brav**
- **Mathematical methods of science, A. S. Tikhomirov**
- **Hamiltonian Mechanics, I. M. Krichever**
- **Symmetric Functions, E. Yu. Smirnov**
- Mathematical Statistics[1], A. V. Klimenko, A. S. Skripchenko
- Markov Chains, A. Dymov

SPRING

- **Geometry and Topology, V. A. Kiritchenko**
- **Basics of Functional Analysis, M. Z. Rovinsky**
- **Advanced Linear Algebra, K. G. Kuyumzhiyan**
- **Calculus of Variations, M. Mariani**
- **Commutative Algebra, A. S. Khoroshkin**

### Advanced Level Courses

These courses are recommend for deep learning the preferred mathematical subjects. In the Table of Contents on p. 2,3, the references to these courses are typesetted by the regular font.

FALL

- **Number Theory, M. V. Finkelberg**
- Analytic Number Theory, A. B. Kalmynin
- **Lie Groups And Lie Algebras, G. I. Olshanski**
- Arithmetical Dynamics[1], C. Favre

SPRING

- **Differential topology, A. A. Gaifullin**
- Analytic Number Theory, A. B. Kalmynin
- **Functional Analysis, A. Yu. Pirkovskii**
- **Dynamics And Ergodic Theory, A. S. Skripchenko, A. V. Zorich**
- **Algebraic Topology, A. G. Gorinov**
- Analysis of several complex variables, A. A. Glutsyuk

[1] The course will be given in the second module (November – December). Learning load: 2 classes per week; worth: 3 credits.

## MSc Math Seminar

This seminar consists of 3 independent sections. Each of them works during two semesters, 1 class per week, and worths 5 credits per semester for the students of MSs programme «Mathematics». Every student of this program has to participate at least one section of the MSc Math seminar each semester[1] of the learning period. Other students can take any section of the MSc Math seminar in any term. In this case it worths 3 credits for the Fall term and 4 credits for the Spring term.

FALL

◦ Geometric structures on manifolds M. S. Verbitsky
◦ Probability and Stohastics, A. V. Kolesnikov, V. Konakov
◦ Functional Analysis and Noncommutative Geometry, A. Yu. Pirkovskii

SPRING

◦ Geometric structures on manifolds M. S. Verbitsky
◦ Probability and Stohastics, A. V. Kolesnikov, V. Konakov
◦ Functional Analysis and Noncommutative Geometry, A. Yu. Pirkovskii

## Other seminars

FALL

◦ Convex and algebraic geometry, V. A. Kiritchenko, E. Yu. Smirnov, A. I. Esterov
◦ Combinatorics of Vassiliev invariants, M. E. Kazarian, S. K. Lando
◦ Representations and Probability[2], A. I. Bufetov, A. Dymov, A. V. Klimenko, M. Mariani, G. I. Olshanski
◦ **Frobenius Manifolds, S. M. Natanzon, P. I. Dunin–Barkowski**
◦ **Homotopy theory, A. G. Gorinov**
◦ Harmonic analysis, A. Yu. Pirkovskii
◦ Integrable Systems, I. Marshall

SPRING

◦ Convex and algebraic geometry V. A. Kiritchenko, E. Yu. Smirnov, A. I. Esterov
◦ Combinatorics of Vassiliev invariants, M. E. Kazarian, S. K. Lando
◦ Representations and Probability[2], A. I. Bufetov, A. Dymov, A. V. Klimenko, M. Mariani, G. I. Olshanski
◦ **Introduction to Symplectic and Contact Geometry, P. E. Pushkar**
◦ Smooth, PL-, and topological manifolds, A. G. Gorinov
◦ Deformation theory with the view of Mori theory, V. S. Zhgoon
◦ Introduction to the theory of integrable equations, A. K. Pogrebkov
◦ Toric Vareties, K. G. Kuyumzhiyan

[1]The sections participated in different semesters may be different.
[2]This seminar is joint with the Independent University of Moscow and the Steklov Math Institute.

## Course descriptions

### primary level course «Advanced Linear Algebra»

**lecturer: K. G. Kuyumzhiyan**

**title: Advanced Linear Algebra**

**learning load: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**description:** This course is aimed to introduce main notions of Linear Algebra and their instances in other fields of mathematics.

**prerequisites:** some notions from the Fall term course «Algebra and Arithmetics» will be used, especially, fields and groups.

**syllabus:**

1. Matrices and Matrix Operations. Systems of Linear Equations. Cramer's Rule.
2. Vector spaces.
3. Linear Transformations.
4. Symmetry and Permutation Representations.
5. Bilinear Forms.
6. Linear Groups.
7. Basics of Representation Theory.

**textbooks:**


- Artin, Michael; Algebra. Prentice Hall, Inc., Englewood Cliffs, NJ, 1991. xviii + 618 pp.
- Vinberg, E. B.; A course in algebra. Translated from the 2001 Russian original by Alexander Retakh. Graduate Studies in Mathematics, 56. American Mathematical Society, Providence, RI, 2003. x + 511 pp
- Lang, Serge; Algebra. Revised third edition. Graduate Texts in Mathematics, 211. Springer-Verlag, New York, 2002. xvi + 914 pp.
- Lang, Serge; Introduction to Linear Algebra. Second Edition. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1986.

**PRIMARY LEVEL COURSE «ALGEBRA AND ARITHMETICS»**

**LECTURER: V. S. Zhgoon**

**TITLE: Algebra and Arithmetics**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** The aim of the course is to give introduction to basic notions of algebra and number theory. We plan to start from the algebraic properties of integer numbers, arithmetic of residues and basic properties of polynomials: such as Chinese remainder theorem, little Ferma's theorem, Wilson's lemma, quadratic residues. This will give us motivation for introducing more general notions in the group theory, commutative and non-commutative algebra. In particular we shall also study basic properties of finite groups such as cosets, normal, nilpotent and solvable subgroups, Sylow theorems, basic notions of commutative algebra: such as ideals, modules, maximal and prime ideals, localization, and basic notions of non-commutative algebra.

**PREREQUISITES:** The course tends to be elementary and very flexible, the program will depend on the listeners. All material required for understanding the course will be explained or reminded.

**SYLLABUS:**

1. Basic notions of integer numbers and residues
2. Chinese remainder theorem, little Ferma's theorem, Wilson's lemma.
3. Quadratic residues. Gauss reciprocity law.
4. Basic notions of group theory. Cosets, normal, nilpotent and solvable subgroups.
5. Group actions. Orbits, stabilizers, normalizers, conjugacy classes. Burside formula.
6. Sylow theorems*.
7. Basic notions of commutative algebra: rings, fields, algebras, ideals, modules.
8. Properties of finite fields.
9. Nilpotence, radicals, maximal and prime ideals, localization.
10. Basic notions of non-commutative algebra. Structure theory for non-commutative algebras.*

**TEXTBOOKS:**


1. E. B. Vinberg, A course in algebra, AMS No. 56, 2003.
2. K. Ireland, M. Rosen. A classical introduction to modern number theory, Springer Science & Business Media Vol. 84, 2013.
3. S. Lang, Algebra, Revised third edition, Graduate Texts in Mathematics 1.211, 2002.
4. D. S. Dummit, R. M. Foote. Abstract algebra, Vol. 3, Hoboken: Wiley, 2004.
5. R. B. Ash, Basic abstract algebra: For graduate students and advanced undergraduates, Courier Corporation, 2013.


**COMMENTS:** Marked by stars are more complicated topics that will be considered only if the time allows.

**PRIMARY LEVEL COURSE «ALGEBRAIC GEOMETRY: A FIRST GEOMETRIC LOOK»**

**LECTURER: V. S. Zhgoon**

**TITLE: Algebraic Geometry: A First Geometric Look**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** Algebraic geometry studies geometric loci looking locally as a solution set for a system of polynomial equations on an affine space. It gives an explicit algebraic explanation for various geometric properties of figures, and in the same time, brings up a geometric intuition underlying abstract purely algebraic constructions. It plays an important role in many areas of mathematics and theoretical physics, and provides the most visual and elegant tools to express all aspects of the interaction between different branches of mathematical knowledge. The course gives the geometric flavor of the subject by presenting examples and applications of the ideas of algebraic geometry, as well as a first discussion of its technical tools.

**PREREQUISITES:** linear and multilinear algebra, basic ideas of polynomials, commutative rings and their ideals, tensor products, affine and projective spaces, topological spaces and their open, closed and compact subsets. No deep knowledge is assumed, all essential definitions and technique will be recalled during the course.

**SYLLABUS:**

- Projective spaces. Geometry of projective quadrics. Spaces of quadrics.
- Lines, conics. Rational curves and Veronese curves. Plane cubic curves. Additive law on the points of cubic curve.
- Grassmannians, Veronese's, and Segre's varieties. Examples of projective maps coming from tensor algebra.
- Integer elements in ring extensions, finitely generated algebras over a field, transcendence generators, Hilbert's theorems on basis and on the set of zeros.
- Affine Algebraic Geometry from the viewpoint of Commutative Algebra. Maximal spectrum, pullback morphisms, Zariski topology, geometry of ring homomorphisms.
- Agebraic manifolds, separateness. Irreducible decomposition. Projective manifolds, properness. Rational functions and maps.
- Dimension. Dimensions of subvarieties and fibers of regular maps. Dimensions of projective varieties.
- Linear spaces on quadrics. Lines on cubic surface. Chow varieties.
- Vector bundles and their sheaves of sections. Vector bundles on the projective line. Linear systems, invertible sheaves, and divisors. The Picard group.
- Tangent and normal spaces and cones, smoothness, blowup. The Euler exact sequence on a projective space and Grassmannian.

**TEXTBOOKS:**


- A. L. Gorodentsev, Algebra II. Textbook for Students of Mathematics. Springer, Ch. 1, 2, 10, 11, 12. Springer, 2017.
- A. L. Gorodentsev, Algebraic Geometry Start Up Course, MCCME.

- J. Harris, Algebraic Geometry. A First Course, Springer.
- D. Mumford, Red book of varieties and schemes, Springer LNM 1358.

**COMMENTS:** This course may be joint with the Math in Moscow program, see http://www.mccme.ru/mathinmoscow/.
The materials for previous versions of this course are available at
http://gorod.bogomolov-lab.ru/ps/stud/projgeom/1718/list.html

**ADVANCED LEVEL COURSE «ALGEBRAIC TOPOLOGY»**

**LECTURER: A. G. Gorinov**

**TITLE: Algebraic Topology**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** One of the main goals of algebraic topology is to answer the question whether two given topological spaces are homeomorphic or homotopy equivalent. This question and several related ones arise not only in topology, but also in mathematical physics, algebra and geometry of any kind. Classical cohomology and generalisations such as K-theory etc are among the the main computational tools that in some cases, allow one to answer this question. This course explains systematically all these tools and their applications. It is intended as a continuation of the primary level course «Introduction to Algebraic Topology».

**PREREQUISITES:** basic algebra (groups, rings, fields), topology (topological and metric spaces, continuous maps, homotopy between continuous maps, coverings and the fundamental group) and category theory (categories, functors and natural transformations). However, all necessary notions will be recalled if required.

**SYLLABUS:**

- Introduction. How to calculate the homology groups of surfaces.
- Singular homology. Basic homological algebra: exact sequences, complexes, 5-lemma, homotopy.
- Homological algebra continued: acyclic models.
- First applications of acyclic models: homotopy invariance and excision for singular cohomology.
- CW-complexes, cellular homology and its particular cases and analogues. Simplicial complexes and simplicial homology. Smooth manifolds, Morse functions, handle decompositions and Morse homology.
- Homology and cohomology with coefficients. The universal coefficient theorems.
- The Künneth isomorphisms.
- Cup and cap products. Topological manifolds and the Poincaré duality.
- Lefschetz theorems. The contribution of a nondegenerate fixed point in the manifold case.
- Vector bundles; the glueing construction. Constructing new bundles using given bundles.
- Chern and Stiefel – Whitney classes. The addition formula.
- The Euler class. Applications of characteristic classes.

**TEXTBOOKS:**

- D. Fuchs, A. Fomenko. A Course in Homotopy Theory.
- A. Hatcher. Algebraic Topology.
  http://www.math.cornell.edu/~hatcher/AT/ATpage.html
- A. Hatcher. Vector bundles and K-theory.
  http://www.math.cornell.edu/~hatcher/VBKT/VBpage.html
- J. Milnor, J. Stasheff. Characteristic classes.

**ADVANCED LEVEL COURSE «ANALYSIS OF SEVERAL COMPLEX VARIABLES»**

**LECTURER: A. A. Glutsyuk**

**TITLE: Analysis of several complex variables.**

**LEARNING LOAD: Spring term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** Analysis of several complex variables is a necessary pre-requisite to study many important domains of contemporary mathematics such as algebraic geometry, complex dynamics, singularity theory, differential equations etc. While holomorphic functions of several complex variables share many basic properties of functions of one variable, new phenomena of analytic extention occurs. For example, they can have neither isolated singularities, nor compact sets of singularities. The statement of Riemann Mapping Theorem in higher dimensions is strongly false. Namely, generic pair of two simply connected domains in complex space are not biholomorphically equivalent. Each complex space of dimension at least two contains a proper domain that is biholomorphically equivalent to the ambient space (Fatou – Bieberbach domain). Theory of holomorphic convexity and Stein manifolds together with basic sheave theory allows to prove important extension and approximation theorems. For example, each holomorphic function on a submanifold of a linear complex space is the restriction to if of a global holomorphic function on the ambient space. The GAGA principle in algebraic geometry says that every analytic object on a complex projective algebraic manifold is algebraic. The cours will cover the above mentioned topics, including basic analytic set theory, biholomorphic automorphisms and introduction to complex dynamics.

**PREREQUISITES:** basic calculus, complex analysis of one variable.

**SYLLABUS:**

1. Holomorphic functions of several complex variables. Cauchy – Riemann equations, Cauchy formula, Osgood Lemma, Taylor series.
2. Convergence of power series and convergence radius. Equivalent definition of holomorphic function.
3. Analytic extension. Erasing singularities. Hartogs Theorem.
4. Analytic sets: Implicit Function Theorem, Weierstrass Preparatory Theorem, factoriality of local ring of holomorphic functions, Weierstrass polynomials in two variables.
5. Analytic sets: decomposition into irreducible components, criterium of irreducibility, local covering presentation and Proper Mapping Theorem (without proof).
6. Introduction to complex algebraic geometry. Chow Theorem. Biholomorphic automorphisms of projective space.
7. Cauchy Inequality. Henri Cartan's theorem on automorphisms of bounded domains tangent to identity.
8. Generalized Maximum Principle and Schwarz Lemma. Automorphisms of ball and polydisk.
9. Introduction to complex dynamics: linearization theorems, polynomial automorphisms of $\mathbb{C}^2$, Fatou-Bieberbach domains.
10. Domains of holomorphy. Holomorphic convexity. Levi convexity. Levi form. Oka's theorems on equivalence of these notions (with proofs of their simple parts). Pseudoconvexity. Riemann domains.
11. Dolbeault cohomology, $\bar{\partial}$-problem, $\bar{\partial}$-Poincare lemma.
12. Cousin problems. Sheaf cohomology. Analytic hypersurfaces in domains of holomorphy as zero loci of global holomorphic functions.

13. Coherent analytic sheaves. Stein manifolds. Extension and approximation theorems. Cartan A and B Theorems (without proof).

**TEXTBOOKS:**


- R. Gunning, H. Rossi, Analytic functions of several complex variables. AMS, 2009.
- B. Shabat. Introduction to complex analysis. Part II: Functions of several variables. Translations of Mathematical Monographs, AMS, 1992.
- Ph. Griffiths, J. Harris. Principles of algebraic geometry, vol. 1. J. Wiley & Sons, 1978.

**ADVANCED LEVEL COURSE «ANALYTIC NUMBER THEORY»**

**LECTURER: A. B. Kalmynin**

**TITLE: Analytic Number Theory**

**LEARNING LOAD: Two semesters of 2018/19 A. Y., 1 class per week, 3 credits per semester**

**DESCRIPTION:** Analytic number theory is an area of number theory that uses analytic methods to study properties of the integers. No progress towards some famous problems such as Golbach's conjecture, Waring's problem or twin primes conjecture would be possible without the development of analytic methods such as bounds for exponential sums and theorems on the distribution of prime numbers. In this course we will study some results concerning averages of certain arithmetical functions (such as divisor function or Euler totient function), prime numbers in arithmetic progressions, properties of the Riemann zeta function and Dirichlet L-functions and exponential sums. We will also see how to use these results to prove certain classical number-theoretical facts. Some of the applications will be straightforward, but we also will learn many unexpected ones. For example, we will deduce Linnik's theorem on seven cubes from Siegel-Walfisz theorem.

**PREREQUISITES:** Complex analysis (basic properties of holomorphic functions, Cauchy's integral formula, Maximum modulus principle, Weierstrass factorization theorem), Analysis ($O$-notation, Lebesgue –Stieltjes integration), Algebra (fundamental theorem of arithmetic)

**SYLLABUS:**

**Fall term:**

1. Arithmetical functions and their averages: an elementary approach. Convolutions of arithmetical functions, Möbius inversion formula, Dirichlet divisor problem, Gauss circle problem, normal and maximal orders of arithmetical functions.

2. Contour integration method. Riemann zeta function, its basic properties and functional equation. Phragmen – Lindelof principle. Prime number theorem, explicit formula, zero-free region for the zeta function. Hardy – Voronoi summation formula.

3. Dirichlet characters, Dirichlet L-functions, Polya – Vinogradov inequality, Page's theorem, Landau – Siegel zeros, Siegel – Walfisz theorem on primes in arithmetic progressions, Linnik's seven cubes theorem.

4. Basic sieve methods, smooth numbers, Selberg sieve, applications.

**Spring term:**

1. The theorem on approximation of a trigonometric sum by a shorter one. Approximate functional equation for the Riemann zeta function. Theory of exponent pairs. Zero density estimates. Primes in short intervals.

2. Estimates for the Weyl sums, equidistribution modulo 1, zero-free regions for zeta function. Waring's problem.

3. Large sieve method and its applications: Linnik's theorem on the least quadratic nonresidue, Brun – Titchmarsh inequality, Galois group of a random polynomial, Selberg's conditional theorem on primes in very short intervals.

**TEXTBOOKS:**

- A. A. Karatsuba, «Basic analytic number theory».
- H. L. Montgomery, R. C. Vaughan, «Multiplicative number theory I: Classical theory».
- A. A. Karatsuba, S. M. Voronin, «The Riemann zeta-function».
- T. Tao, «Analytic prime number theory» https://terrytao.wordpress.com/category/teaching/254a-analytic-prime-number-theory/.

**ADVANCED LEVEL COURSE «ARITHMETICAL DYNAMICS»**

**LECTURER: C. Favre**

**TITLE: Arithmetical Dynamics**

**LEARNING LOAD:**

**DESCRIPTION: , module 1 (September–October), 1 class per week, 2 credits. classes per week, 5 credits per semester.** These lectures are aimed at presenting some aspects of the dynamics of polynomials in one variable with coefficients in a number field. We shall discuss two deep conjectures that have been proved to be very influential in the development of this field: the uniform boundedness conjecture by Silverman, and the dynamical Andre–Oort conjecture Baker and DeMarco. It will be the opportunity to present various tools coming from arithmetic geometry or from complex dynamics that are used to approach these challenging problems.

**PREREQUISITES:** elementary notions in non-Archimedean analysis in one variable are welcome (e.g. A. Robert «A Course in $p$-adic Analysis», chapters 1, 2, and Sections 6.1, 6.2).

**SYLLABUS:**

- The uniform boundedness conjecture
- Fatou/Julia theory for complex polynomials
- Canonical heights for complex polynomials
- Non-archimedean polynomial dynamics
- The dynamical Andre–Oort conjecture
- Equidistribution of points of small heights

**PRIMARY LEVEL COURSE «BASICS OF FUNCTIONAL ANALYSIS»**

**LECTURER: M. Z. Rovinsky**

**TITLE: Basics of functional analysis**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** As its name suggests, Functional analysis originates from the study of functions. The fundamental idea here is to interpret functions as points in an appropriate vector space and to study analytic problems in terms of mappings between such spaces. However, as soon as considered vector spaces are infinite-dimensional, nontrivial results arise only after vector spaces are provided with a nontrivial topology and the mappings are supposed to be continuous. This course proposes an introduction to methods and basic results of Functional analysis, both abstract and related to various spaces of functions.

**PREREQUISITES:** Acquaintance with Linear Algebra and some basic Analysis is required. Some knowledge of measure theory would be helpful.

**SYLLABUS:**

- Normed fields and vector spaces
- Banach spaces and their examples
- Hahn–Banach theorem (and extension of bounded linear functionals)
- Uniform boundedness principle
- Bounded inverse theorem, open mapping theorem, closed graph theorem
- Hilbert spaces
- Spectral theory
- Fourier transform on commutative locally compact groups

**TEXTBOOKS:**

1. Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, 1978.
2. Dirk Werner, Funktionalanalysis, Springer-Verlag, 2005.
3. A. Weil, L'intégration dans les groupes topologiques et ses applications, Hermann, 1953.

**COMMENTS:** Marked by stars are more complicated topics that will be considered only if the time allows.

**PRIMARY LEVEL COURSE «CALCULUS OF VARIATIONS»**

**LECTURER: M. Mariani**

**TITLE: Calculus of Variations**

**LEARNING LOAD: Spring term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** The lectures provide an introduction to Calculus of Variations, addressing both classical subjects (action functionals, isoperimetric problems), and modern approaches (direct methods, applications to physics and optimal control). The student will be required to understand the theoretical aspects of the theory, as well as to apply it to specific cases.

**PREREQUISITES:** Mathematical Analysis, Elementary general Topology, Basic Functional Analysis.

**SYLLABUS:**

1. Historical model problems and preliminaries: convex analysis, Sobolev spaces.
2. Classical methods: Euler-Lagrange equations, optimal control, the Hamiltonian approach, viscosity solutions, applications.
3. Direct methods: basic theory, elliptic problems (existence, uniqueness, regularity), Euler-Lagrange revisited, relaxation of integral functionals, applications.

**TEXTBOOKS:**

- Bernard Dacorogna; Introduction to the calculus of variations; Imperial College Press, 3rd ed (2014).
- Bruce van Brunt; The Calculus of Variations; Springer (2004).
- Israel M. Gelfand, Sergey V. Fomin; Calculus of Variations; Dover (1963) [Selected topics].
- Michael Struwe; Variational Methods; Springer (2008) [Selected topics].
- Mariano Giaquinta, Stefan Hildebrandt; Calculus of Variations I; Springer (2004) [Selected topics].

**COMMENTS:** Depending on the number and interests of students, one of the following topic can be addressed in some additional lectures: optimal control, minimal surfaces, homogenization.

**PRIMARY LEVEL COURSE «CLASSICAL ANALYSIS AND ODE»**

**LECTURER: T. Takebe**

**TITLE: Classical Analysis and ODE**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** The calculus (differentiation and integration) is one of the most fundamental tools in mathematics. In this course, after reviewing definitions of differentiation and integration of functions of several variables, the properties of such functions will be discussed together with basics on ordinary differential equations (ODE).

**PREREQUISITES:** Basic calculus (functions of one variable and elementary properties of real numbers).

**SYLLABUS:**

- Review of calculus of functions in one variable.
- Differentiation of functions of many variables: partial derivatives and total differential.
- Inverse function theorem and implicit function theorem.
- Integration along curves.
- Integration of functions of many variables.
- Ordinary differential equations: basic examples and various methods for solving them.
- Fundamental theorems on ordinary differential equations: existence and uniqueness of solutions.

**TEXTBOOKS:**

- W. Rudin. Principles of mathematical analysis.
- J. Munkres, Analysis on Manifolds.
- E. Hairer, G. Wanner. Analysis by its history.

**COMMENTS:** The final score will be based on the results of quiz during the course and the final exam.

**PRIMARY LEVEL COURSE «COMMUTATIVE ALGEBRA»**

**LECTURER: A. S. Khoroshkin**

**TITLE: Introduction to Commutative Algebra**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** At its most basic level, algebraic geometry is the study of the geometry of solution sets of polynomial systems of equations. Classically, the coefficients of the polynomial equations are assumed to lie in an algebraically closed field. Considering more general coefficient rings, in particular rings of integers in number fields, one arrives at modern algebraic geometry and algebraic number theory. Commutative algebra provides the tools for answering basic questions about solutions sets of polynomial systems, such as finite generation of the system, existence of solutions in some extension of the coefficient ring, dimension and irreducible components, and smoothness and singularities.

**PREREQUISITES:** Basic courses given at the faculty of mathematics for the first 3 semesters, including (a) basic algebra (groups, rings, fields), (b) Linear algebra (tensor products), (3) Basic geometry

**SYLLABUS:**

- Rings and ideals
- Modules
- Integral dependence
- Localization
- Primary decomposition
- Dedekind domains
- Dimension theory
- Tensor products
- Length

**TEXTBOOKS:**

- M. Reid, «Undergraduate commutative algebra», CUP, 1995.
- M. Atiyah, «Introduction to commutative algebra.», Westview press, 1994. Russian translation: М. Атья, И. Макдоналд, «Введение в коммутативную алгебру», М.: Мир, 1972.
- G. Kemper, «A course in commutative algebra», Springer, 2010.
- D. Eisenbud. «Commutative Algebra: With a View Toward Algebraic Geometry», NY: Springer-Verlag, 1999.

**ADVANCED LEVEL COURSE «DIFFERENTIAL GEOMETRY»**

**LECTURER: O. V. Schwarzman**

**TITLE: Differential geometry**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** The course will serve as an introductory guide to basic topics of Differential and Riemann geometry: the theory of Riemannian and Loretzian manifolds, the theory of affine connections on manifolds, The Gauss – Bonne – Chern – Weil theory.

**PREREQUISITES:** курс рассчитан на студентов-математиков старших курсов бакалавриата, а также магистрантов и аспирантов.

**SYLLABUS:**

- **Differential and Riemann geometry of smooth hypersurfaces in the Euclidean space.**
  - Parallel transport. The Gauss Map. The Shape operator.
  - The metric connection. Covariant derivatives. Parallel transport.
  - Completeness and geodesics. The Exponential Map. The Hopf – Rinow theorem.
  - Curvature. Geodesics.
- **Riemann manifolds:** Riemannian metric and Levi – Chivita connection.
- **Curvature.**
  - Calculations with curvature tensor. The Gauss curvature.
  - The Ricci tensor.
  - Spaces of constant curvature.
- **Variational theory of geodesics.**
  - First and second variation of arc length.
  - Jacobi's equation and conjugate points.
  - The Gauss lemma and polar coordinates.
- **Connections in vector bundles.**
  - Parallel transport and Covariant derivatives.
  - Introduction to the Chern – Weil theory.

**TEXTBOOKS:**

1. Gallot, Hulin, Lafontaine, Riemannian Geometry.
2. Milnor, Morse Theory.

**ADVANCED LEVEL COURSE «DIFFERENTIAL TOPOLOGY»**

**LECTURER: A. A. Gaifullin**

**TITLE: Differential topology**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** We plan to discuss two topics, which are central in topology of smooth manifolds, the $h$-cobordism theorem and theory of characteristic classes. The $h$-cobordism theorem proved by S. Smale in 1962 is the main (and almost the only) tool for proving that two smooth manifolds (of dimension $\geq 5$) are diffeomorphic. In particular, this theorem implies the high-dimensional Poincaré conjecture (for manifolds of dimensions $\geq 5$). Characteristic classes, in particular, Pontryagin classes are very natural invariants of smooth manifolds. Computation of characteristic classes can help one to distinguish between non-diffeomorphic manifolds. We plan to finish the course with the theorem by J. Milnor on non-trivial smooth structures on the 7-dimensional sphere. This theorem is based both on methods of Morse theory and theory of characteristic classes.

**PREREQUISITES:** Differential and algebraic topology, Morse theory, theory of characteristic classes.

**SYLLABUS:**

1. De Rham cohomology. Singular homology. Pairing between homology and cohomology.
2. Multiplication in cohomology and intersection of cycles. Poincaré duality.
3. Morse functions. Cobordisms corresponding to critical points. Morse inequalities. Lefschetz theorem on hyperplane sections.
4. Smale's $h$-cobordism theorem. High-dimensional Poincaré conjecture.
5. Principal bundles and their characteristic classes. Chern – Weil theory. Chern classes and Pontryagin classes.
6. Integral Chern classes and Pontryagin classes.
7. Smooth structures on the 7-dimensional sphere.

**TEXTBOOKS:**


1. R. Bott, L. W. Tu, Differential forms in algebraic topology.
2. B. A. Dubrovin, A. T. Fomenko, S. P. Novikov, Modern Geometry — Methods and Applications. Part II: The Geometry and Topology of Manifolds.
3. J. Milnor, Morse theory.
4. J. Milnor, Lectures on the $h$-cobordism theorem.
5. J. Milnor, On manifolds homeomorphic to the 7-sphere, Annals of Mathematics, 64 (1956), 399–405.
6. J. Milnor, J. Stasheff, Characteristic classes.
7. S. P. Novikov, I. A. Taimanov, Modern Geometric Structures and Fields.

**PRIMARY LEVEL COURSE «DYNAMICS AND ERGODIC THEORY»**

**LECTURERS: A. S. Skripchenko, A. V. Zorich**

**TITLE: Dynamical Systems in Modern Geometry, Topology and Physics.**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** Dynamical systems in our course will be presented mainly not as an independent branch of mathematics but as a very powerful tool that can be applied in geometry, topology, probability, analysis, number theory and physics. We consciously decided to sacrifice some classical chapters of ergodic theory and to introduce the most important dynamical notions and ideas in the geometric and topological context already intuitively familiar to our audience. As a compensation, we will show applications of dynamics to important problems in other mathematical disciplines. We hope to arrive at the end of the course to the most recent advances in dynamics and geometry and to present (at least informally) some of results of A. Avila, A. Eskin, M. Kontsevich, M. Mirzakhani, G. Margulis.

**PREREQUISITES:** basic differential geometry, topology and measure theory.

**SYLLABUS:**

- Rotation of the circle and continued fractions.
- Introduction to hyperbolic geometry. Möbius transformations. Fuchsian groups.
- Geodesics on surfaces of negative curvature. The geodesic flow and its properties.
- Geodesic flow on modular curve as a continued fraction map.
- Teichmüller space. Teichmüller geodesic flow.
- Interval exchange transformations (IET) as natural generalizations of continued fractions.
- IET as the first return maps on transversal for measured foliations on oriented surface. Poincaré recurrence theorem.
- Key ergodic properties: minimality, ergodicity, number of invariant measures (illustrated by IET).
- Multiplicative ergodic theorem. Topological interpretation of Lyapunov exponents.
- Anosov and Pseudoanosov diffeomorphisms of surfaces. Introduction to hyperbolic dynamics (Markov partitions, invariant measures etc).

**TEXTBOOKS:**

- F. Dal'bo, Geodesic and Horocyclic trajectories, Springer Urtext (2011).
- S. Katok, Fuchsian groups, University of Chicago Press, 1992 (на русском: М.: Факториал, 2002)
- W. Thurston, Geometry and topology of three-manifolds, Princeton University Press, 1997 (на русском: М.: МЦНМО, 2001)
- Ya. Sinai, Introduction to ergodic theory Princeton University Press, 1977 (на русском: изд. Ереванского унив., 1973)
- M. Viana, Ergodic Theory of Interval Exchange Maps, Revista Mathematica Com- plutense 19:1 (2006) 7–-100.

- J.-C. Yoccoz, Interval exchange maps and translation surfaces,
  http://www.college-de-france.fr/media/jean-christophe-yoccoz/UPL15305_PisaLecturesJCY2007.pdf
- G. Margulis, Number theory and homogeneous dynamics,
  http://jointmathematicsmeetings.org/meetings/national/jmm/margulis_colloq_lect_08.pdf
- D. Ruelle, Chaotic Evolution and Strange Attractors, CUP, 1989.

**ADVANCED LEVEL COURSE «FUNCTIONAL ANALYSIS»**

**LECTURER: A. Yu. Pirkovskii**

**TITLE: Functional Analysis**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** Functional analysis studies normed and topological infinite-dimensional vector spaces and representations of algebraic structures on them. It is commonly used in differential equations, analytical and differential geometry, representation theory and quantum physics. The classical areas are the spectral theory of linear operators, distribution theory, operator algebra theory etc. Among relatively new are the noncommutative geometry à la Connes, operator space theory (AKA «quantum functional analysis»), and locally compact quantum groups. We plan to discuss selected topics from the syllabus below. The choice will depend on the material of preceding course «Introduction to Functional Analysis» in order to minimize intersections.

**PREREQUISITES:** the course «Introduction to Functional Analysis», Fall 2018.

**SYLLABUS:**

**Duality for Banach spaces.** Dual spaces and dual operators. The duals of classical Banach spaces. The canonical embedding into the bidual. Reflexivity. Annihilators, preannihilators. The duals of subspaces and quotients. Relations between properties of operators and their duals.

**Elementary spectral theory.** The spectrum of an algebra element. Banach algebras. The nonemptiness and the compactness of the spectrum. The Gelfand – Mazur Theorem. Spectral radius. The point spectrum, the continuous spectrum, and the residual spectrum of a linear operator. Spectra and duality. Calculations for classical operators.

**Compact and Fredholm operators.** Basic properties, examples. The Fredholm index. The Riesz – Schauder theory. The Fredholm Alternative. Properties of the spectrum of a compact operator. Applications to integral equations. The Nikolski – Atkinson criterion. The Calkin algebra. The continuity of the index. The stability of the index under compact perturbations. The essential spectrum. Applications to Toeplitz operators.

**Compact operators on a Hilbert space.** The Hilbert – Schmidt theorem on the diagonalization of compact selfadjoint operators. The Schmidt theorem on the structure of compact operators. Applications to the Sturm – Liouville problem. Hilbert – Schmidt operators and nuclear operators. The trace.

**Topological vector spaces.** Locally convex spaces. Examples. Continuous linear operators. Normability and metrizability criteria. Dual pairs and weak topologies. The bipolar theorem and corollaries. The Banach – Alaoglu theorem. Weak topologies and compact operators.

**Distributions.** Operations on distributions. The sheaf of distributions. The support. Compactly supported distributions and tempered distributions. Tensor product and convolution of distributions. Structure theorems for distributions. The Fourier transform of tempered distributions.

**Commutative Banach algebras.** Maximal ideals and characters of a commutative Banach algebra. The closedness of maximal ideals. The Gelfand spectrum. The Gelfand transform. $C^*$-algebras. The Gelfand – Naimark theorem on commutative $C^*$-algebras.

**The Spectral Theorem.** The continuous and Borel functional calculi for a selfadjoint operator. Positive operators. The polar decomposition. Spectral measures and representations of algebras of continuous functions. The Spectral Theorem and the functional model for a selfadjoint operator. Multiplicity theory and classification of selfadjoint operators.

**TEXTBOOKS:**


1. A. Ya. Helemskii, Lectures and exercises on functional analysis. AMS, 2006.
2. V. I. Bogachev, O. G. Smolyanov, Real and functional analysis (in Russian), R&CD, 2009.
3. J. B. Conway. A course in functional analysis, Springer, 1990.
4. G. J. Murphy. $C^*$-algebras and operator theory, Academic Press, 1990.

**PRIMARY LEVEL COURSE «GALOIS THEORY»**

**LECTURER: C. Brav**

**TITLE: Galois theory**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** Galois theory is the study of roots of polynomials and their symmetries in terms of Galois groups. As the algebraic counterpart of the fundamental group of topology, the Galois group is an essential object in algebraic geometry and number theory.

**PREREQUISITES:** Basic algebra: groups, rings, linear algebra over a field.

**SYLLABUS:**

- Review of polynomial rings and more general principal ideal domains.
- Extensions of fields, algebraic and transcendental.
- Splitting fields of polynomials and Galois groups.
- The fundamental theorem of Galois theory.
- Computing Galois groups.
- Applications.

**TEXTBOOKS:** James Milne. Fields and Galois Theory,
http://www.jmilne.org/math/CourseNotes/FT.pdf

**PRIMARY LEVEL COURSE «GEOMETRY AND TOPOLOGY»**

**LECTURER: V. A. Kiritchenko**

**TITLE: Geometry and Topology**

**LEARNING LOAD: Spring term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** The course covers basic notions of topology such as metric spaces, smooth manifolds and fundamental group. The main focus is on concrete examples such as Riemann surfaces, projective spaces, Grassmannians. The course can serve as an introduction to more advanced geometry and topology courses.

**PREREQUISITES:** Basic Abstract Algebra (groups, rings and vector spaces)

**SYLLABUS:**

- Reminder on set theory: countable and uncountable sets, Cantor set, axiom of choice, non-measurable set.
- Point-set topology: topological spaces, open and closed subsets, continuous functions, homeomorphism and homotopy equivalence.
- Metric spaces, compactness, manifolds. Peano curve and Cantor staircase function.
- Fundamental group and covering spaces, Riemann surfaces.
- Projective spaces and Grassmannians.

**TEXTBOOKS:** J. R. Munkres. Topology — A First Course. Prentice Hall, 1975

**COMMENTS:** The course is suitable for all 1st year M.Sc. students or 2nd year B.Sc. students.

**ADVANCED LEVEL COURSE «HAMILTONIAN MECHANICS»**

**LECTURER: I. M. Krichever**

**TITLE: Hamiltonian Mechanics**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** This is one of the basic theoretical physics courses for students in their 3–4 year of undergraduate studies and for Masters students. A core of mathematical methods of modern theory of Hamiltonian systems are concepts created in various branches of mathematics: the theory of differential equations and dynamical systems; the theory of Lie groups and Lie algebras and their representations; the theory of smooth maps of manifolds. Many modern mathematical theories, such as symplectic geometry and theory of integrable systems have arisen from problems of classical mechanics. That's why this course is recommended not only for those who plan to continue their studies in «Mathematical Physics» master program, but also for those who are planing to continue pure mathematical studies.

**PREREQUISITES:** No physics courses prerequisites.

**SYLLABUS:**

1)Lagrangian formalism: Least action principle; Euler – Lagrange equations; first integrals and symmetries of action.

2) Basics of Hamiltonian formalism: phase space; Legendre transform; Poisson brackets and symplectic structure; Darboux thoerem, Hamiltonian equayions.

3)Examples: Geodesics on Lie groups. Mechanics of solid body and hydrodinamics of ideal fluid.

4) Separations of variables and integrability: Hamitonian-Jacobi equations; canonical transformations. Moment map. Arnold – Liouville integrable systems. Lax representation.

**TEXTBOOKS:**
**Литература для подготовки**


[1] Л. Д. Ландау, Е. М. Лифшиц. Курс теоретической физики, т.1, Механика. М.: Наука, 1988.

[2] В. И. Арнольд. Математические методы классической механики. 3-е изд. М.: Наука, 1989.

[3] O. Babelon, D. Bernard, M. Talon. Introduction to Classical Integrable Systems. CUP, 2003.

**ADVANCED LEVEL COURSE «INTEGRABLE SYSTEMS AND ADS/CFT CORRESPONDENCE»**

**LECTURER: Mikhail Alfimov**

**TITLE: Integrable Systems and AdS/CFT Correspondence**

**LEARNING LOAD: Spring term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** In the present course the foundations of the AdS/CFT correspondence and integrable structures appearing in this context will be explained for the case of 4-dimensional superconformal gauge theory. As a pedagogical examples of the integrable systems solved by the Bethe Ansatz method Bose gas and Principal Chiral Field models will be considered. There will be given an introduction into the applications of the theory of integrable systems to the study of the spectrum of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and dual superstring theory on the $AdS_5 \times S^5$ background. The course is intended for PhD and Master students. Postdocs and Bachelor students are also welcome.

**PREREQUISITES:** The course is best suitable for PhD and Master students.

**SYLLABUS:**

- The model of Bose gas. Bethe equations for the spectrum of the Bose gas model and their thermodynamic limit. Thermodynamic Bethe Ansatz (TBA) equations for the Bose gas model.
- Principal Chiral Field (PCF) Model. Bethe equations for the spectrum of the PCF model and their thermodynamic limit. TBA equations for the PCF.
- Y- and T-system (Hirota equations) for PCF. Wronskian solution of the Hirota equations.
- AdS/CFT correspondence. String background $AdS_5 \times S^5$ as the solution of the supergravity equations.
- Classical integrability of the PCF model and $AdS_5 \times S^5$ superstring $\sigma$-model.
- Derivation of the S-matrix for the superstring $\sigma$-model on $AdS_5 \times S^5$ from Zamolodchikov-Faddeev algebra.
- Bethe equations for the XXX Heisenberg spin chain (1-loop spectrum of anomalous dimensions of the local operators in the $\mathfrak{sl}(2)$ sector of $\mathcal{N} = 4$ SYM). Asymptotic Bethe equations for the spectrum of $\mathcal{N} = 4$ SYM. TBA equations for the spectrum of $\mathcal{N} = 4$ SYM.
- Y- and T-system for the spectrum of $\mathcal{N} = 4$ SYM and the corresponding Hirota equations. Wronskian solution of these equations.
- Derivation of the AdS/CFT Quantum Spectral Curve for $AdS_5 \times S^5$ superstring theory and $\mathcal{N} = 4$ SYM.
- Application of the QSC method for the $\mathfrak{sl}(2)$ sector of $\mathcal{N} = 4$ SYM. Non-perturbative characteristics of the operator trajectories of the operator trajectories in the $\mathcal{N} = 4$ SYM.

**TEXTBOOKS:**

**[1]** V. Korepin, N. Bogoliubov, A. Izergin, Quantum Inverse Scattering Method and Correlation Functions, CUP, 1993, https://doi.org/10.1017/CBO9780511628832.

**[2]** N. Beisert, C. Ahn, L. F. Alday et al. Lett Math Phys (2012) 99:3, https://doi.org/10.1007/s11005-011-0529-2.

**[3]** E. D'Hoker, D. Z. Freedman, Supersymmetric Gauge Theories and the AdS/CFT correspondence. Strings, Branes and Extra Dimensions, 2004, https://doi.org/10.1142/9789812702821_0001.

**COMMENTS:** In the Spring term of 2017, I was giving the similar course at Skolkovo. In the next Fall term, it was moved to the Faculty of Mathematics. According to my previous experience, the actual duration of classes will be rather 2–2.5 hours than the traditional 1.5 hours. The course will also include solving the problems by the students. Keywords: supersymmetric gauge theory, integrable systems, gauge/string duality.

**PRIMARY LEVEL COURSE «INTRODUCTION TO COMBINATORIAL THEORY»**

**LECTURER: Yu. M. Burman**

**TITLE: Introduction to Combinatorial Theory**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** Combinatorics is a part of mathematics studying finite sets. The question to answer is usualy «how many»: how many are there connected graphs with $n$ numbered vertices containing no cycles? how many are there ways to draw diagonals of a convex $n$-gon so as to cut it into triangles? etc. This question is answered by a multitude of methods from real and complex analysis, number theory, geometry, and more. We do not expect, however, that the students are familiar with all these areas: the necessary techniques will be explained in the course. Combinatorics is very rich in applications, ranging from mathematical physics to algebraic geometry to finance, including topology and dynamical systems on the way. Very often questions from various sciences eventually turn to be combinatorial problems. Combinatorics is an indispensable part of every mathematician's education.

**PREREQUISITES:** no.

**SYLLABUS:** In the course, we study various combinatorial objects and various methods of solution of combinatorial problems. For convenience we list methods and objects separately in the program; in the actual course we alternate them.

1. Methods.
    - Formal power series.
    - Linear recurrence.
    - Lagrange inversion theorem.
    - Transfer matrix.
2. Objects.
    - Binomial coefficients.
    - Lattice paths.
    - Catalan numbers.
    - Partitions and compositions.
    - Trees.
    - Parking functions.
    - Hurwitz numbers.
    - Tutte polynomial.

**TEXTBOOKS:**


- S. Lando, *Lectures on Generating Functions*, AMS Student Mathematical Library, V. 23, 2003.
- G. F. Andrews, *The Theory of Partitions*, Cambrisdge University Press, 1984 (reprinted in 2003).

**ADVANCED LEVEL COURSE «LIE GROUPS AND LIE ALGEBRAS»**

**LECTURER: G. I. Olshanski**

**TITLE: Lie Groups and Lie Algebras**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** We shall begin with the basics of the theory of Lie groups and Lie algebras. Then we shall provide an accessible introduction to the theory of finite-dimensional representations of classical groups on the example of the unitary groups $\mathrm{U}(N)$.

**PREREQUISITES:** standard course of linear algebra.

**SYLLABUS:**

- linear Lie groups and their Lie algebras
- universal enveloping algebras
- the Haar measure on a linear Lie group
- general facts about representations of compact groups and their characters
- radial part of Haar measure
- Weyl's formula for characters of the unitary groups
- Weyl's unitary trick
- classification and realization of representations.

**TEXTBOOKS:**

1. W. Fulton, J. Harris, Representation theory. Springer 1991; Russian translation: 2017.
2. J. Faraut, Analysis on Lie groups. An introduction. CUP, 2008.
3. A. Kirillov, Jr., Introduction to Lie groups and Lie algebras. CUP, 2008;
   https://www.math.stonybrook.edu/~kirillov/mat552/liegroups.pdf

**PRIMARY LEVEL COURSE «MARKOV CHAINS»**

**LECTURER: A. Dymov**

**TITLE: An introduction to Markov chains and their applications.**

**LEARNING LOAD: Fall term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** Markov chains form the simplest class of random processes for which the future does not depend on the past but depends only on the present state of the process. Being rather simple, at the same time Markov chains have very deep and beautiful mathematics. They are known as probably the most important class of random processes, in particular, because of the numerous applications in mathematics, physics, biology, economics, etc. Indeed, once a stochastic process is given, it is natural to simplify it by assuming that the future does not depend on the past, and very often this approximation works well. The present course is the introduction to the theory of Markov chains. It will concern with their most important properties and the most known applications. The course is aimed at the 3rd and 4th year students, but is also possible for 1st and 2nd year students.

**PREREQUISITES:** basic courses of analysis and linear algebra.

**SYLLABUS:**

1. Examples of models leading to Markov chains.
2. Markov chains with finite number of states.
3. Ergodic properties of Markov Chains.
4. Applications of Markov Chains: random walks, birth-death processes, etc.
5. Entropy of Markov Chains.
6. Markov Chains with infinite number of states.

**TEXTBOOKS:**


- A. N. Shiryaev, Probability, 2nd ed., Springer, New-York (1995).
- J. G. Kemeny, J. L. Snell, Finite Markov chains, Springer-Verlag (1976).
- B. V. Gnedenko, Theory of probability, 6th ed., Boca Raton, FL: CRC Press (1998).

**PRIMARY LEVEL COURSE «MATHEMATICAL METHODS OF SCIENCE»**

**LECTURER: A. S. Tikhomirov**

**TITLE: Mathematical methods of science**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** The notion of vector, respectively, principal bundle over a smooth manifold is one of the central notions in modern mathematics and its applications to mathematical and theoretical physics. In part icular, all known types of physical interations (gravitational, electromagnetic, etc.) are described in terms of connections and other geometric structures on vector/principal bundles on underlying manifolds. The properties of physical fields can be formulated in terms of geometric invariants of connections such as curvature and characteristic classes of corresponding vector/principal bundles. In this course we give an introduction to the differential geometry of vector and principal bundles and consider their metrics, connections, curvature and characteristic classes. Some applications to algebraic geometry, topology, and gauge theory of classical fields (in particular, Maxwell equations, Yang–Mills theory) are discussed.

**PREREQUISITES:** the standard courses in algebra, calculus, geometry, and topology for the first year of undergraduate study.

**SYLLABUS:**

- **Manifolds.** Manifolds and smooth maps. Tangent vectors. Vector fields. Differential forms. Exterior differentiation on a manifold. Exterior differentiation on $\mathbb{R}^3$. Pullback of differential forms.
- **Riemannian Manifolds.** Inner products on a vector space. Riemannian metric. Existence of a Riemannian metric. Regular curves. Arc length parametrization. Signed curvature of a plane curve. Orientation and curvature.
- **Affine Connections.** Affine connections. Torsion and curvature. The Riemannian connection.
- **Vector Bundles.** Definition of a vector bundle. The vector space of sections. Extending a local section to a global section. Local operators. Restriction of a local operator to an open subset. Frames. F-linearity and bundle maps. Multilinear maps over smooth functions.
- **Connections on a Vector Bundle.** Connections on a vector bundle. Existence of a connection on a vector bundle. Curvature of a connection on a vector bundle. Riemannian bundles. Metric connections. Restricting a connection to an open subset. Connections at a point.
- **Connection, curvature, and torsion forms.** Connection and curvature forms. Connections on a framed open set. Metric connection relative to an orthonormal frame. Connections on the tangent bundle. Covariant differentiation along a curve. Connection-preserving diffeomorphisms. Christoffel symbols.
- **Geodesics.** The definition of a geodesic. Reparametrization of a geodesic. Existence of geodesics. Geodesics in the Poincaré half-plane. Parallel translation. Existence of parallel translation along a curve. Parallel translation on a Riemannian manifold.
- **Exponential maps.** The exponential map of a connection. The differential of the exponential map. Normal coordinates. Left-invariant vector fields on a Lie group. Exponential map for a Lie group. Naturality of the exponential map for a Lie group. Adjoint representation. The exponential map as a natural transformation.
- **Distance and volume.** Distance in a Riemannian manifold. Geodesic completeness. Dual 1-forms under a change of frame. The volume form in local coordinates.

- **Operations on vector bundles.** Vector subbundles. Subbundle criterion. Quotient bundles. The pullback bundle. Examples of the pullback bundle. The direct sum of vector bundles. Other operations on vector bundle.

- **Vector-valued forms.** Vector-valued forms as sections of a vector bundle. Products of vector-valued forms. Directional derivative of a vector-valued function. Exterior derivative of a vector-valued form. Differential forms with values in a Lie algebra. Pullback of vector-valued forms. Forms with values in a vector bundle. Tensor fields on a manifold. The tensor criterion.

- **Connections and curvature again.** Connection and curvature matrices under a change of frame. Bianchi identities. The first Bianchi identity in vector form. Symmetry properties of the curvature tensor. Covariant derivative of tensor fields. The second Bianchi identity in vector form. Ricci curvature. Scalar curvature. Defining a connection using connection matrices. Induced connection on a pullback bundle.

- **Characteristic classes.** Invariant polynomials on $\mathfrak{gl}_r(\mathbb{R})$. The Chern–Weil homomorphism. Characteristic forms are closed. Differential forms depending on a real parameter. Independence of characteristic classes of a connection. Functorial definition of a characteristic class.

- **Pontrjagin classes. The Euler class and Chern classes.** Vanishing of characteristic classes. Pontrjagin classes. The Whitney product formula. Orientation on a vector bundle. Characteristic classes of an oriented vector bundle. The Pfaffian of a skew-symmetric matrix. The Euler class. Generalized Gauss–Bonnet theorem. Hermitian metrics. Connections and curvature on a complex vector bundle. Chern classes.

- **Some applications of characteristic classes.** The generalized Gauss–Bonnet theorem. Characteristic numbers. The cobordism problem. The embedding problem. The Hirzebruch signature formula. The Riemann–Roch.

- **Principal bundles.** Principal bundles. The frame bundle of a vector bundle. Fundamental vector fields of a right action. Integral curves of a fundamental vector field. Vertical subbundle of the tangent bundle $TP$. Horizontal distributions on a principal bundle.

- **Connections on a principal bundle.** Connections on a principal bundle. Vertical and horizontal components of a tangent vector. The horizontal distribution of an Ehresmann connection. Horizontal lift of a vector field to a principal bundle. Lie bracket of a fundamental vector field. Horizontal distributions on a frame bundle. Parallel translation in a vector bundle. Horizontal vectors on a frame bundle. Horizontal lift of a vector field to a frame bundle. Pullback of a connection on a frame bundle under a section.

- **Curvature on a principal bundle.** Curvature form on a principal bundle. Properties of the curvature form. The associated bundle. The fiber of the associated bundle. Tensorial forms on a principal bundle. Covariant derivative. A formula for the covariant derivative of a tensorial form.

- **Characteristic classes of principal bundles.** Invariant polynomials on a Lie algebra. The Chern–Weil homomorphism.

- **Applications to gauge theory of classical fields.** The Yang–Mills functional. Maxwell equations, rank two Euclidean Yang–Mills theory: instantons.

**TEXTBOOKS:**

- L. Tu. Differential Geometry: Connections, Curvature, and Characteristic Classes. Springer, 2017.

- K. Nomizu. Lie Groups and differential geometry. 1956.

- R. Palais. The geometrization of physics. 1981.

- T. Aubin. Nonlinear analysis on manifolds, Monge-Ampere equations. Springer, 1982.

**Additional textbooks:**

- C. H. Taubes. Differential geometry: bundles, connections, metrics and curvature. Oxford, 2011.
- P. Petersen. Riemannian geometry. Springer, 2006.
- J. Milnor, J. Stasheff. Characteristic classes. Princeton, 1974.
- C. Nash, S. Sen. Topology and geometry for physicists. Academic Press, 1987.

**PRIMARY LEVEL COURSE «MATHEMATICAL STATISTICS»**

**LECTURERS: A. V. Klimenko, A. S. Skripchenko**

**TITLE: Introduction to mathematical statistics**

**LEARNING LOAD: second module (November–December) of the Fall term 2018/19 A.Y., 2 classes per week, 3 credits.**

**DESCRIPTION:** The main goal of mathematical statustics is an adaptation of the theoretical probabilistic models to some practical problems in economics, physics, medicine, social sciences. Typically the precise distribution or random process that describes some phenomenon is not known; however, some information can be extracted from the series of observations or repeated experiments; this data is be used to select the most appropriate model.

**PREREQUISITES:** the most basic part of probability theory, like distributions of random variables, mathematical expectations and variances for random variables, statement of the central limit theory (knowledge of the proof is not required).

**SYLLABUS:**

- Statistical models, samples, descriptive statistics. Empirical approach: empirical distribution and Glivenko – Cantelli theorem.
- Parametric statistics: estimations and their main properties. Unbiased estimators. Efficient estimators. Cramer – Rao low bound. Consistent estimators. Fisher – Neumann factorization theorem. Rao – Blackwell theorem. Confidence intervals.
- Statistical hypothesis testing. Common test statistics. Null hypothesis statistical significance testing. Neumann – Pearson lemma and the most powerful test at the given significance level.

**TEXTBOOKS:**

- David Freedman, Robert Pisani, Roger Purves, «Statistics».
- David Williams, «Weighing the odds: a course in probability and statistics».
- М. Лачугин, «Наглядная математическая статистика».

**COMMENTS: It is a short (1-module) course given in the *second* module (November–December).** The course is strongly recommended for the students who are planning to include econometrics and related subjects in their future individual plans. The course can be taught in English or in Russian.

**ADVANCED LEVEL COURSE «NUMBER THEORY»**

**LECTURER: M. V. Finkelberg**

**TITLE: *p*-adic numbers**

**LEARNING LOAD: Fall term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** $p$-adic numbers were discovered by Hensel at the turn of 20-th century as an alternative to the classical real analysis in order to solve number theoretic problems. The development of $p$-adic analysis allowed Dwork to prove the first Weil conjecture on the number of solutions of algebraic equations over finite fields in 1960's. The goal of the course is to understand this proof and to learn the basics of $p$-adic analysis.

**PREREQUISITES:** первый год бакалавриата (стандартные курсы алгебры, анализа, геометрии, комбинаторики и топологии) и теория Галуа.

**SYLLABUS:**

- Metrics on the field of rational numbers.
- $p$-adic numbers.
- Hilbert symbol.
- Quadratic forms over $p$-adic fields.
- Minkowski – Hasse theorem.
- Algebraic closure of $p$-adic field.
- Tate field.
- Artin – Hasse exponent.
- Newton polygons.
- Zeta-functions.
- Rationality of zeta-function.

**TEXTBOOKS:**


**[К]** Н. Коблиц, «$p$-адические числа, $p$-адический анализ и дзета-функции».

**[С]** Ж. П. Серр, «Курс арифметики».

**PRIMARY LEVEL COURSE «SYMMETRIC FUNCTIONS»**

**LECTURER: E. Yu. Smirnov**

**TITLE: Symmetric functions**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** The theory of symmetric functions is one of the central branches of algebraic combinatorics. Being a rich and beautiful theory by itself, it also has numerous connections with the representation theory and algebraic geometry (especially geometry of homogeneous spaces, such as flag varieties, toric and spherical varieties). In this course we will mostly focus on the combinatorial aspects of the theory of symmetric functions and study the properties of Schur polynomials. In representation theory they appear as characters of representations of $GL(n)$; they are also closely related with the geometry of Grassmannians. The second half of the course will be devoted to Schubert polynomials, a natural generalization of Schur polynomials, defined as «partially symmetric» functions. Like the Schur functions, they also have a rich structure and admit several nice combinatorial descriptions; geometrically they appear as representatives of Schubert classes in the cohomology ring of a full flag variety. Time permitting, we will also discuss K-theoretic (non-homogeneous) analogues of Schur and Schubert polynomials.

**PREREQUISITES:** the standard courses in algebra and discrete mathematics or combinatorics. Some knowledge of representation theory of symmetric and general linear groups is not required, but helpful.

**SYLLABUS:**

1. Symmetric polynomials. The ring of symmetric functions. Bases of the ring of symmetric functions: elementary, complete, monomial symmetric functions, power sums. Transition formulas between these bases.
2. Schur functions. Algebraic definition. Jacobi-Trudi formula. Combinatorial definition, equivalence with the algebraic definition. Young tableaux.
3. Applications to combinatorics: enumerating plane partitions. MacMahon formula.
4. Richardson – Schensted – Knuth (RSK) correspondence. Jeu de taquin.
5. Multiplication of Schur functions. Pieri rule. Littlewood – Richardson rule. Knutson – Tao puzzles*.
6. Symmetric group, its Coxeter presentation. The Bruhat order. The Lehmer code and the essential set of a permutation.
7. «Partially symmetric» polynomials. Divided difference operators. Schubert polynomials.
8. Properties of Schubert polynomials. Monk's formula, Lascoux transition formula.
9. Combinatorial presentation of Schubert polynomials. Pipe dreams. Positivity. Fomin – Kirillov theorem.
10. Flagged Schur functions, determinantal formulae.
11. Generalizations*: double Schubert polynomials, Stanley symmetric functions, Grothendieck polynomials.

**TEXTBOOKS:**


1. W. Fulton. Young tableaux, With Applications to Representation Theory and Geometry. CUP, 1997

2. L. Manivel. Fonctions symétriques, polynômes de Schubert et lieux de dégénérescence. Société Mathématique de France.

3. I. G. Macdonald. Symmetric functions and Hall polynomials. 2nd edition. Clarendon Press, 1998.

4. R. Stanley. Enumerative combinatorics, vol.2. CUP, 1999.

**COMMENTS:** In the Syllabus, marked with stars are advanced topics that will be considered if the time allows. There are Russian translations for the textbooks [1], [3], [4], and English for [2].

**PRIMARY LEVEL COURSE «TOPICS IN DIFFERENTIAL AND ALGEBRAIC TOPOLOGY»**

**LECTURER: P. E. Pushkar**

**TITLE: Topics in differential and algebraic topology**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** We will start by introducing the basic notions and objects such as manifolds and tangent spaces. After that we will discuss the inverse and implicit function theorems from the viewpoint of tangent spaces, Sard's lemma and the Morse lemma, as well as Whitney's immersion and embedding theorems. In the topology part of the seminar we will see CW-complexes, approximation theorems, simplicial homology, the fundamental group and higher homotopy groups. We will also discuss the de Rham cohomology which forms a connection between the differential and topological approaches.

**PREREQUISITES:** Undergraduate calculus and linear algebra.

**SYLLABUS:** Tentative list of topics:

A. Differential topology:

- A.0 Smooth manifolds and tangent bundles
- A.1 Inverse and implicit function theorems
- A.2 Sard's lemma
- A.3 Morse lemma
- A.4 Whitney's immersion and embedding theorems
- A.5 Transversal intersections and transversality theorems

B. Algebraic topology:

- B.1 CW-complexes
- B.2 Cellular and simplicial approximation theorems
- B.3 Complexes and exact sequences
- B.4 Simplicial homology
- B.5 The fundamental group and higher homotopy groups
- B.6 De Rham cohomology

**TEXTBOOKS:**


- V. I. Arnold, S. M. Gusein-Zade, A. N. Varchenko. Singularities of Differentiable Maps, Vol. 1. Classification of Critical Points, Caustics and Wave Fronts.
- R. Bott, L. Tu. Differential forms in algebraic topology.
- J. Milnor, J. Stasheff. Characteristic classes.

## SEMINAR DESCRIPTIONS

### STUDENT RESEARCH SEMINAR «CONVEX AND ALGEBRAIC GEOMETRY»

**ADVISORS: A. I. Esterov, V. A. Kiritchenko, E. Yu. Smirnov**

**TITLE: Convex and algebraic geometry**

**LEARNING LOAD: Two semesters of 2018/19 A. Y., 1 class per week, 3 credits per semester**

**DESCRIPTION:** Our research seminar is devoted to the many connections between convex and algebraic geometry. This interaction has many important applications in various areas of mathematics: combinatorics, representation theory, mathematical physics to name a few. One of the most well-known examples is the combinatorial description of the toric varieties in terms of polytopes. Yet another recent and up-to-date application is the theory of Newton – Okounkov bodies. Participants will tell about classical topics as well as recent papers that they find important on

http://arxiv.org/find/grp_math/1/AND+cat:+math.AG+all:+polytope/0/1/0/all/0/1
http://arxiv.org/find/grp_math/1/AND+cat:+math.RT+all:+polytope/0/1/0/all/0/1

providing extensive background material for those less familiar with the subject.

**PREREQUISITES:** Accessible to geometrically oriented 2nd year students. No prerequisites required beyond the mandatory courses from the 1st year (and the fall of the 2nd year, for the spring term). Introduction to algebraic geometry is a plus, but not required.

**SYLLABUS:**

- Convexity and lattices.
- Smooth convex bodies.
- Convex polyhedra.
- Mixed volumes.
- Convex inequalities.
- Ehrhart polynomials.
- Patchworking.
- Bernstein – Kushnirenko Theorem.
- Fiber polytopes and polyhedral subdivisions.
- Tropical and Enumerative Geometry.
- Coxeter groups and polytopes.
- Number of faces of a convex polytope.
- Gelfand – Zetlin polytopes and Schubert calculus.

**TEXTBOOKS:**

- Gelfand I., Kapranov M., Zelevinsky A., Discriminants, Resultants, and Multidimensional Determinants, 1994.
- Maclagan D., Sturmfels B., Introduction to Tropical Geometry, 2015
- Viro O., Itenberg I., Patchworking Algebraic Curves Disproves the Ragsdale Conjecture, 1996
- G. Ziegler, Lectures on Polytopes, 1995, https://books.google.ru/books?id=xd25TXSSUcgC&lpg=PP1&hl=ru&pg=PP1#v=onepage&q&f=false

**STUDENT RESEARCH SEMINAR «COMBINATORICS OF VASSILIEV INVARIANTS»**

**ADVISORS: M. E. Kazarian, S. K. Lando**

**TITLE: Combinatorics of Vassiliev invariants**

**LEARNING LOAD: Two semesters of 2018/19 A. Y., 1 class per week, 3 credits per semester**

**DESCRIPTION:** This students' research seminar is devoted to combinatorial problems arising in knot theory. The topics include finite order knot invariants, graph invariants, matroids, delta-matroids, integrable systems and their combinatorial solutions. Hopf algebras of various combinatorial species are studied. Seminar's participants give talks following resent research papers in the area and explaining results of their own.

**PREREQUISITES:** no.

**SYLLABUS:**

1. Knots and their invariants.
2. Knot diagrams and chord diagrams.
3. 4-term relations for chord diagrams, graphs, and delta-matroids.
4. Weight systems.
5. Constructing weight systems from Lie algebras.
6. Hopf algebras of graphs, chord diagrams and delta-matroids.
7. Combinatorial solutions to integrable hierarchies.
8. Khovanov homology

**TEXTBOOKS:**

1. S. Chmutov, S. Duzhin, Y. Mostovoy. CDBook. CUP, 2012.
2. S. Lando, A. Zvonkin. Graphs on Surfaces and Their Applications. Springer, 2004.

**STUDENT RESEARCH SEMINAR «DEFORMATION THEORY WITH THE VIEW OF MORI THEORY»**

**ADVISOR: V. S. Zhgoon**

**TITLE: Deformation theory with the view of Mori theory**

**LEARNING LOAD: Spring term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** The aim of the course is to give the introduction to the advanced topics of algebraic geometry which are usually omitted in the standard course. The fruitfull schematic approach of Grothendieck allows to construct such objects as: Hilbert scheme (which classifies the subschemes of a given scheme), Quot scheme and scheme of morphisms between two schemes. The deformation theory studies the infinitesimal structure of these schemes, that allows to say much about the objects where the deformations are considered. This idea was used by Mori who used the geometry of rational curves to study the birational geometry of projective varieties. In the course we shall discuss these variety of topics.

**PREREQUISITES:** Basic notions of algebraic geometry of schemes or a good knowledge of commutative algebra and basic algebraic geometry.

**SYLLABUS:**

1. Deformation theory. Deformations of different objects: schemes, sheaves, morphisms etc. Tangent spaces to the space of deformations. Infinitesimal obstructions.
2. Hilbert, Quot, Hom and Chow schemes.
3. Applications to the spaces of rational curves. Bend and break technique.
4. Multiplier ideals. Kawamata-Viehweg vanishing theorem. Shokurov non-vanishing and base-point-freeness theorem. Mori cone theorem.
5. Fulton – Hansen connectedness theorem and its applications to geometry of projective varieties. Zak theorems.

**TEXTBOOKS:**


1. Lazarsfeld, Robert K. Positivity in algebraic geometry I: Classical setting: line bundles and linear series. Vol. 48. Springer, 2004.
2. Hartshorne, Robin. Deformation theory. Vol. 257. Springer, 2009.
3. Debarre, Olivier. Higher-dimensional algebraic geometry. Springer, 2013.
4. Kollár, János. Rational curves on algebraic varieties. Vol. 32. Springer, 2013.
5. Esnault, Hélène, and Eckart Viehweg. Lectures on vanishing theorems. DMV seminar. 1992.
6. Matsuki, Kenji. Introduction to the Mori program. Springer, 2013.

**STUDENT RESEARCH SEMINAR «FROBENIUS MANIFOLDS»**

**ADVISORS: S. M. Natanzon, P. I. Dunin–Barkowski**

**TITLE: Frobenius Manifolds, Cohomological Field Theories and Topological Recursion**

**LEARNING LOAD: Spring term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** The theory of Frobenius – Dubrovin manifolds connects the theory of singularities, integrable systems, differential geometry, topological invariants of manifolds, operads, moduli spaces of algebraic curves, mirror symmetry etc. Analytical aspects of the theory were developed by Dubrovin about 20 years ago. Algebraic and topological aspects are described by cohomological field theories discovered by Kontsevich and Manin. After that the theory of Frobenius – Dubrovin sees active development and plays an important role in many branches of mathematics and mathematical physics. In the course, we will discuss the equvalence of various seemingly different definitions of Frobenius – Dubrovin manifolds: flat deformations of Frobenius algebras, orthogonal coordinate systems, pencils of flat cometrics, solutions of WDVV equations, cyclic operads, cohomological field theories and so on, and consider various nontrivial examples of such manifolds arising naturally on spaces of versal deformations of singularities, spaces of orbits of Coxeter groups, Hurwitz spaces, in the theory of differential equations of the hydrodynamic type, theory of Gromov – Witten invariants, etc. In addition, we will discuss the theory of spectral curve topological recursion, and its connections to some of the aforementioned objects and theories.

**PREREQUISITES:** Basics of differential geometry.

**SYLLABUS:**

1. Frobenius algebras.
2. WDVV equation.
3. Frobenius manifolds.
4. Classification of 2D and 3D Frobenius manifolds.
5. Hurwitz – Frobenius spaces.
6. Basics of singularity theory and its connection to the theory of Frobenius manifolds.
7. Cohomological field theories and Gromov – Witten invariants.
8. Integrable systems related to Frobenius manifolds.
9. Spectral curve topological recursion and its connection to cohomological field theories; Givental's formalism.

**TEXTBOOKS:**


1. B. Dubrovin, «Geometry of 2D topological field theories», Springer, Lect. Notes in Math., 1620 (1996), 120–348.
2. S. M. Natanzon, «Geometry of two-dimensional topological field theories».
3. Yu. I. Manin, «Frobenius Manifolds, Quantum Cohomology, and Moduli Spaces», AMS Colloquium Publications, 1999.
4. P. I. Dunin–Barkowski, N. Orantin, S. Shadrin, L. Spitz «Identification of the Givental formula with the spectral curve topological recursion procedure», Comm. Math. Phys. 328 (2014), 669-–700.

## STUDENT RESEARCH SEMINAR «GEOMETRIC STRUCTURES ON MANIFOLDS»

## (A SECTION OF THE OBLIGATORY MATH SEMINAR OF MSC PROGRAM «MATHEMATICS»)

**ADVISOR: M. S. Verbitsky**

**TITLE: Geometric structures on manifolds**

**LEARNING LOAD: two semesters, 5 credits per semester, 1 class per week.**

**DESCRIPTION:** Students of HSE give talks about current problems of algebraic and differential geometry.

**PREREQUISITES:** Analysis on manifolds, complex analysis, differential geometry.

**SYLLABUS:** Every week new speakers are chosen, with a new topics of their choice.

**TEXTBOOKS:**

A. Besse, «Einstein manifolds»

A. S. Mishchenko, «Vector bundles and their applications»

M. Gromov, «Metric Structures for Riemannian and Non-Riemannian spaces»

P. Gauduchon, «Calabi's extremal Kähler metrics: An elementary introduction»

N. J. Hitchin, A. Karlhede, U. Lindström, M. Roček, «Hyperkähler metrics and supersymmetry», Comm. Math. Phys. **108** (1987), 535–589.

**STUDENT RESEARCH SEMINAR «HARMONIC ANALYSIS»**

**ADVISOR: A. Yu. Pirkovskii**

**TITLE: Harmonic analysis and unitary representations**

**LEARNING LOAD: Fall term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** Harmonic analysis on groups and unitary representation theory are closely related areas of mathematics, complementary to each other. They play an important role in analysis, geometry, topology, physics, and other fields of science. In essence, they grew out of two classical topics that are usually studied by undergraduate students in mathematics. The two topics are the theory of trigonometric Fourier series and the representation theory (over $\mathbb{C}$) of finite groups. Among other things, we plan to explain what the above topics have in common, what the representation theory of compact groups looks like, what the Tannaka – Krein duality is, and what all this has to do with the Fourier transform. We are also going to construct harmonic analysis on locally compact abelian groups. This theory includes the Pontryagin duality and generalizes the Fourier transform theory on the real line. As an auxiliary material, the basics of Banach algebra theory will also be given.

**PREREQUISITES:** The Lebesgue integration theory and the basics of functional analysis. Some knowledge of the representation theory of finite groups will also be helpful.

**SYLLABUS:**

1. Introduction. A toy example: harmonic analysis on a finite abelian group. Classical examples: harmonic analysis on the integers, on the circle, and on the real line.
2. The main objects: topological groups; the Haar measure; a relation between the left and right Haar measures; unitary representations; the general Fourier transform.
3. Banach algebras: the $L^1$-algebra of a locally compact group; the spectrum of a Banach algebra element; commutative Banach algebras, the Gelfand spectrum, the Gelfand transform; basics of $C^*$-algebra theory; the $C^*$-algebra of a locally compact group; the 1st (commutative) Gelfand – Naimark theorem.
4. Locally compact abelian groups: the dual group; the Fourier transform as a special case of the Gelfand transform; the Plancherel theorem; the Pontryagin duality.
5. Compact groups: the averaging procedure; irreducible representations are finite-dimensional; decomposing unitary representations into irreducibles; the Peter – Weyl theorem; the orthogonality relations; the Fourier transform and its inverse; the Plancherel theorem; the Tannaka – Krein duality.

**TEXTBOOKS:**


1. A. Deitmar, S. Echterhoff. Principles of harmonic analysis. Springer, 2009.
2. G. B. Folland. A course in abstract harmonic analysis. CRC Press, 1995.
3. G. J. Murphy. $C^*$-algebras and operator theory. Academic Press, 1990.
4. D. P. Zhelobenko. Principal structures and methods of representation theory. MCCME, 2004 (in Russian). English transl.: AMS, 2006.
5. A. Robert. Introduction to the representation theory of compact and locally compact groups. Cambridge University Press, 1983.
6. K. H. Hofmann, S. Morris. The structure of compact groups. Walter de Gruyter, 2006.
7. A. Joyal, R. Street. An introduction to Tannaka duality and quantum groups. Lecture Notes in Math. 1488, 411–492. Springer, 1991.

**STUDENT RESEARCH SEMINAR «HOMOTOPY THEORY»**

**ADVISOR: A. G. Gorinov**

**TITLE: Homotopy theory**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** We give an introduction to generalised cohomology and stable homotopy theory. At first, we consider examples and a few applications of generalised homology and cohomology, such as the Bott periodicity, Hopf invariant 1, complex structures on spheres, representing classes by manifolds, cobordism rings. After that we develop a general theory: spectra, stable homotopy category, fibration and cofibration sequences, the Whitehead theorem, the Atiyah duality.

**PREREQUISITES:** Basic algebraic topology as covered, e.g., in [3] or [2, Ch.1-2]. However, the material of [3, Ch.4] and [2, Ch.1] will be recalled if necessary.

**SYLLABUS:**

1. Axioms for generalised (co)homology.
2. Cofibration sequences for spaces. Omega-spectra and cohomology theories.
3. Fibration sequences for spaces.
4. First applications: the Dold – Thom theorem, representing rational homotopy classes by manifolds.
5. Brown's representability theorem for cohomology.
6. Basic K-theory.
7. Complex Bott periodicity; extending the complex K-theory to a cohomology theory.
8. Applications of K-theory: the Hopf invariant 1 and almost complex structures on spheres.
9. Spectra and stable homotopy category. Homotopy groups of spectra.
10. Thom spectra and cobordism. The Pontrjagin – Thom theorem.
11. Calculation of $\pi_*(\mathrm{MO})$ and $\pi_*(\mathrm{MSO}) \otimes \mathbb{Q}$.
12. Whitehead's theorem for spectra.
13. Spectra can be desuspended.
14. Fibration and cofibration sequences for spectra.
15. Duality for spectra. The Alexander duality.
16. The Thom isomorphism for generalised cohomology and the Atiyah duality.
17. The topological Riemann – Roch theorem and applications. Schwarzenberger's conditions on the Chern numbers of complex vector bundles on $\mathbb{CP}^n$.

**TEXTBOOKS:**


1. J. Adams, Stable Homotopy and Generalised Homology.
2. D. Fuchs, A. Fomenko, A course in Homotopy Theory.
3. A. Hatcher, Algebraic Topology.

**STUDENT RESEARCH SEMINAR «INTEGRABLE SYSTEMS OF CLASSICAL MECHANICS»**

**ADVISOR: I. Marshall**

**TITLE: Integrable Systems of Classical Mechanics**

**LEARNING LOAD: Fall term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** Integrable systems are Hamiltonian systems possessing a complete set of commuting integrals. Since the Kepler problem, which is perhaps the most important physical model in history, and throughout the development of classical mechanics and celestial mechanics, they make recurrent appearances. In modern times they continue to find applications in diverse areas of physics and mathematics, and their study involves many interesting mathematical techniques. In this course we will treat a series of examples by means of which we shall encounter some of the various methods and approaches used in the subject.

**PREREQUISITES:** It will be useful if students have some knowledge of classical mechanics, and already have some familarity with Lagrangian mechanics. If they have studied Hamiltonian systems, so much the better. Material from the 2nd year course «Calculus on Manifolds» will be sufficient for most of the technical parts of the course. Fundamental aspects will be revised during the first few seminars, and this course may be expected to be useful for the consolidation of concepts from all of the «prerequisites».

**SYLLABUS:**

1. First examples: Jacobi problem of geodesics on an ellipsoid, Neumann problem of harmonic oscillators on a sphere, Euler problem of rigid body motion, Kepler problem, KdV equation.
2. Review of differential geometry: smooth manifold, tangent and cotangent bundles, vector-fields and $p$-forms, exterior derivative, Lie derivative, symplectic structure, Poisson structure.
3. Darboux Theorem, generating functions, Liouville Theorem.
4. Euler, problem of two centres: Elliptic coordinates on $\mathbb{R}^n$, Lax representation, Garnier and Calogero – Moser systems.
5. The KdV story, and a superficial look at inverse scattering.
6. Lie groups, Lie algebras.
7. Involution theorems, the $r$-matrix, Toda models, Kowalevski top, Manakov top.
8. Hamiltonian reduction, examples — Calogero and others.

**TEXTBOOKS:**

- Reyman, Semenov–Tian-Shansky. «Integrable Systems» (in Russian).
- Bableon, Talon «Integrable Systems» (in English).

**ADVANCED LEVEL COURSE «INTRODUCTION TO SYMPLECTIC AND CONTACT GEOMETRY»**

**LECTURER: P. E. Pushkar**

**TITLE: An Introduction to Symplectic and Contact geometry**

**LEARNING LOAD: Fall term of 2018/19, 2 classes per week, 5 credits per semester.**

**DESCRIPTION:** The course is centered on the notion of symplectic structure, contact structure, legendrian and contact manifolds. In each direction we start from the very basic definition.

**PREREQUISITES:** Introductory courses on differential manifolds and ordinary differential equations.

**SYLLABUS:**

A. Symplectic geometry
A.0 Linear symplectic geometry
A.1. Symplectic structure, Hamiltonian fields, Darboux theorem
A.2 Symplectic reduction.
A.3. Lagrangian manifolds, Maslov index.
A.4. First steps of symplectic topology (Morse theory, Arnold's conjectures, idea of Floer Homology).
A.5* Liouville – Arnold theorem, moment map. The Atiyah – Guillemin – Sternberg convexity theorem.

B. Contact geometry
B.1. Distributions, integrability, contact structures. Examples. One jets of functions.
B.2 Legendrian manifolds.
B.4. Contact geometry and first-order partial differential equations.
B.3. Differential geometry of plane curves and contact structure.
B.4 Generating families.
B.5. An introduction to Lagrangian and Legendrian singularities.

**TEXTBOOKS:**

Arnold V. I., Givental A. B. Symplectic geometry.
Arnold V. I. Mathematical Methods of Classical Mechanics.
Arnold, V. I., Gusein–Zade, S. M., Varchenko, A. N. Singularities of Differentiable Maps, Volume 1. Classification of Critical Points, Caustics and Wave Fronts.

**STUDENT RESEARCH SEMINAR «INTRODUCTION TO THE THEORY OF INTEGRABLE EQUATIONS»**

**ADVISOR: A. K. Pogrebkov**

**TITLE: Introduction to the Theory of Integrable Equations**

**LEARNING LOAD: Spring term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** Creation and development of the theory of integrable equations is one of main achievements of the mathematical physics of the fall of the previous century. In our times ideas and results of this theory penetrate in many branches of the modern mathematics: from string theory to the theory of Riemann surfaces.

**PREREQUISITES:** analysis of one and several real variables, theory of complex variables, linear algebra, theory of linear partial differential equations.

**SYLLABUS:** Commutator identities on associative algebras; $\bar{\partial}$-problem and dressing operators; Lax pairs; Kadomtsev – Petviashvili equation; Soliton solutions of the KP equation; Two-dimensional reduction: KdV equation; Details of the Inverse scattering transform for KdV equation; Soliton solutions of the KdV equations, their properties; dispersion relation and integrals of motion; IST as canonical transformation.

**TEXTBOOKS:** Presentation of the theory of the KdV equation is based on the textbooks:

- S. Novikov, S. V. Manakov, L. P. Pitaevskij, V. E. Zakharov. Theory of solitons. The inverse scattering methods. Contemporary Soviet Mathematics, 1984.
- F. Calogero, A. Degasperis. Spectral Transform and solitons, I. 1982.

Presentation of other topics is based on the current publications.

**COMMENTS:** reading course

**STUDENT RESEARCH SEMINAR «PROBABILITY AND STOHASTICS»**

**ADVISORS: A. V. Kolesnikov, V. Konakov**

**TITLE: Probability Theory with Analytical and Economical Applications.**

**LEARNING LOAD: Two semesters of 2018/19 A. Y., 1 class per week, 3 credits per semester**

**DESCRIPTION:** We discuss all kind of problems related to probabilistic methods in analysis and various applications. The discussed topics cover a broad area and vary every year. The content highly depends on the interest of invited lectureres and participating students.

**PREREQUISITES:** standard linear algebra and analysis, ordinary differential equations. Some experience in functional analysis and stochastics is desirable.

**SYLLABUS:** List of some regularly discussed topics.

- Stochastic differential equations with applications in finance
- Random matrices
- Convex geometry and probability
- Probabilistic and economic applications of the Monge-Kantorovich problem and other extremal problems
- Martingale theory, its financial applications
- Probability distributions on Lie groups
- Stochastic Riemannian geometry
- Infinite-dimensional distributions, Gaussian measures
- Elements of the game theory
- Physical methods in economics
- Levy processes

**STUDENT RESEARCH SEMINAR «REPRESENTATIONS AND PROBABILITY»**

**ADVISOR: A. I. Bufetov (Steklov Math. Inst.), A. Dymov, A. V. Klimenko, M. Mariani, G. I. Olshanski**

**TITLE: Representations and Probability (joint with Steklov Math. Inst. and IUM)**

**LEARNING LOAD: Two semesters of 2018/19 A. Y., 1 class per week, 3 credits per semester**

**DESCRIPTION:** The seminar is mostly aimed to 3–4th year bachelor students, as well as master and PhD students. Senior participants are expected to deliver a talk on the seminar. The seminar topics are the mix of modern results in areas related to representations and probability theory, and older areas, which are prerequisites to the former, as well as keep their own value.

**PREREQUISITES:** Standard courses of calculus, algebra, and probability.

**SYLLABUS:** Tentative topics for fall semester:

- Continuous-time Markov chains and their asymptotical behavior.
- Empirical and invariant measures for Markov chains. Potential theory for Markov chains.
- Determinantal point processes. Results connecting them with Markov chains.
- Large-time behavior of diffusion process. Applications to non-equilibrium statistical mechanics.

Tentative topics for spring semester:

- Classical representations theory.
- Representations of infinite-dimensional groups
- Their connections with algebraic combinatorics (symmetric functions), classical analysis (orthogonal polynomials) and probabilities theory (point processes and Markov dynamics).

**TEXTBOOKS:**


[1] I. I. Gikhman, A. V. Skorokhod. Introduction to the theory of random processes. Dover 1996 (Русский оригинал: И. И. Гихман, А. В. Скороход. Введение в теорию случайных процессов)

[2] S. Kuksin, A. Shirikyan. Mathematics of two-dimensional turbulence. CUP, 2012.

[3] A. Borodin, G. Olshanski. Representations of the infinite symmetric group. CUP, 2017.


**COMMENTS:** Seminar is held at the Steklov Mathematical Institute in fall semester and at HSE in spring semester. Semesters can be taken independently.

**STUDENT RESEARCH SEMINAR «SMOOTH, PL-, AND TOPOLOGICAL MANIFOLDS»**

**ADVISOR: A. G. Gorinov**

**TITLE: Smooth, PL and topological manifolds**

**LEARNING LOAD: Spring term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** Suppose we are given a topological manifold $X$. When does it admit a smoothing? In other words, when does there exist a smooth manifold $Y$ which is homeomorphic to $X$? And if there does, then how many, up to diffeomorphism? Surprisingly, the answer to these and similar questions can be given in terms of homotopy classes of maps into certain classifying spaces. The aim of the seminar is to provide an introduction to these topics. In particular, we will construct examples of topological manifolds that admit no PL structures and of PL manifolds that admit no smooth structures.

**PREREQUISITES:** smooth manifolds, basic algebraic topology as covered, e.g., by A. Hatcher's «Algebraic topology»; some knowledge of Morse theory and surgery theory would be useful, but we will recall everything we will need.

**SYLLABUS:**

- Microbundles.
- The Kister – Mazur theorem; the tangent bundle of a topological manifold.
- PL manifolds and their tangent bundles.
- Classifying spaces for PL bundles.
- Obstructions to smoothing a PL manifold. The homotopy groups of PL/DIFF are the groups of homotopy spheres.
- Smoothing handles.
- The product structure theorem.
- Milnor's smoothing theorem.
- TOP/PL $= K(\mathbb{Z}/2, 3)$ (a sketch).
- Examples

**TEXTBOOKS:** smoothings of piecewise linear manifolds by A. Hirsch and B. Mazur, foundational essays on topological manifolds, smoothings and triangulations by R. Kirby and L. Siebenmann, topics in geometric topology from lecture notes by J. Lurie.

**STUDENT RESEARCH SEMINAR «TORIC VARETIES»**

**ADVISOR: K. G. Kuyumzhiyan**

**TITLE: Toric Varieties**

**LEARNING LOAD: Spring term of 2018/19, 1 class per week, 3 credits per semester**

**DESCRIPTION:** This is an introduction to the theory of toric varieties, which are algebraic manifolds obtained from convex polytopes by means of some wonderful explicit geometric construction. For example, the standard $n$-simplex gives in this way the projective space of dimension $n$. The advantage of the pass from polytopes to toric varieties is that the crutial combinatorial and geometric properties of polytopes predetermine the key properties of the corresponding varieties and vice versa. Almost all essential algebraic, geometric, and topological characteristics of a toric variety are explicitly computable in terms of its polytope. This makes the toric varieties very suitable for testing algebro-geometric and topological conjectures, seeking examples and counterexamples, etc.

**PREREQUISITES:** In the first half of the course, we need Convex Geometry, Commutative Algebra, and basic properties of affine and projective algebraic varieties. In the second half, some more advanced notions from algebraic geometry are required, such as divisors and algebraic group actions. However, a deep knowledge of Algebraic Geometry is not assumed, and all necessary things will be precisely formulated and either proven or provided with explicit references to textbooks.

**SYLLABUS:** Affine and projective toric varieties, orbit – cone correspondence, automorphisms of affine toric varieties and locally nilpotent derivations, resolution of singularities in dimension 2 and continued fractions. Divisors on toric varieties. Cohomology groups of nonsingular toric varieties.

**TEXTBOOKS:**


- D. Cox, J. Little, H. Schenck. Toric varieties. GTM 124, AMS, 2011.
- W. Fulton. Introduction to toric varieties. Ann of Mathematics Studies 131, Princeton University Press, 1993.
- В. И. Данилов. Геометрия торических многообразий. УМН 33:2(200), 1978, с. 85—134.
- T. Oda. Convex bodies and algebraic geometry. An introduction to the theory of toric varieties. Results in Mathematics and Related Areas (3) 15, Springer-Verlag 1988.


**COMMENTS:** The course is suitable for students of the third year and above.