Seminar work 2. February 9

February 7, 2022

Problem 1. Find the convergence domains of the power series obtained from the series below by opening the brackets and putting monomials in lexicographic order:

a) $\sum_{k=1}^{\infty} k(z_1^2 + 4z_2^2)^k$.
b) $\sum_{k=0}^{\infty} 2^{-k}(z_1 z_2 + z_3^3)^k$.
The Taylor series at the origin of the functions:
c) $\frac{\sqrt{1+z_1+2z_2}}{1+z_1}$.
d) (Homework) $\ln(1 + z_1 + z_2 z_3)\sqrt{1 + z_1 z_2}$.

Problem 2. Find the Taylor series at the origin of the functions

a) $((1 - z_1)(1 - z_2)\dots(1 - z_n))^{-1}$.
b) $((1 - z_1)(1 - 2z_2)\dots(1 - nz_n))^{-1}$.
c) $\ln(1 - z_1)\dots\ln(1 - z_n)$.
d) $\exp(z_1 + \dots + z_n)$.

Problem 3. Find the partial derivatives of the above functions a), b), c) at the origin.

Problem 4. Find the function whose Taylor series at the origin is $\sum_{k,n\geq 1} knz_1^k z_2^n$.

Problem 5. (Homework) Prove that a holomorphic function on a connected domain in $\mathbb{C}^n$ vanishing on a positive measure set is identically zero.