

Homological Method in Quantum Field Theory

Sketch of the course:

This course introduces basic ideas and various recent mathematical developments about quantization that arises from quantum field theory and string theory. The focus is on homological method and its applications in geometry and topology.

The course is addressed to senior and postgraduate students of Mathematics and Physics interested in studying the methods and ideas of modern Mathematical and Theoretical Physics and related topics in Mathematics. The prerequisites are Linear Algebra, Calculus, basic ideas from Differential Equations and Differential Geometry. Some acquaintance with Homological Algebra and Topology is advisable, but not necessary.

Lecturer: Prof. Si Li, YMSC, Tsinghua University



Bio: Si Li (李思) got his Ph.D. in mathematics from Harvard University in 2011. He is currently a professor at Yau Mathematical Sciences Center (YMSC), Tsinghua University. He works on algebraic and geometric aspects of quantum field theory and string theory.

Lecture time:

March 16 - May 25 (Wednesdays) 20:00-21:30 Beijing time (15:00 - 16:30 Moscow time)

March 9, 16, 23, 30;

April 6, 13, 20, 27;

May 11, 18, 25

Join Zoom Meeting:

<https://zoom.us/j/85422463838?pwd=ZmtCOEExMEFmT21jeWY1QUZ0Y3VFQT09>

Meeting ID: 854 2246 3838

Passcode: 073743

Syllabus:

Tentative syllabus about the subjects to be covered in this course:

1. Perturbative theory and Feynman diagram
2. Homotopy algebras and transfer
3. Maurer-Cartan equation and moduli
4. BRST-BV formalism
5. Renormalization and effective field theory
6. Deformation quantization
7. Topological quantum mechanics and index theory
8. 2d chiral conformal field theory and BV quantization
9. Chiral homology and chiral index
10. Topological B-model on Calabi-Yau geometry

11*. Holomorphic Chern-Simons: Large N method

12*. Boundary-Bulk and boundary transfer

13*. Koszul duality and holography

* The last three topics are tentative, to be covered if time permits.

Donaldson Invariants and Hitchin Moduli Space

Sketch of the course:

Donaldson invariants are important invariants to classify the smooth structures of 4-manifolds and were developed by Donaldson in 80s based on the analysis of the moduli space of anti-self-dual instantons. When reducing the instanton equation on the product of a surface with a 2-dimensional torus, one gets the Hitchin equation. The study of the moduli space of the Hitchin equation initiated the study of the non-abelian Hodge theory, which involves the representation theory of the fundamental group and the harmonic Higgs bundle. The study of the non-abelian Hodge theory has been studied for a long time since the middle of 1980s. At the beginning of this century, Hitchin moduli space became a key object in the geometrical Langlands program proposed by Kapustin-Witten. The geometrical Langlands duality conjecture says the brane category of a Hitchin moduli space related to a group G is equivalent to the Hitchin moduli space associated to the dual group GT . This course will provide a quick review of the above important theories and conjectures. This is an advanced course for senior undergraduate students, graduate students or researchers who have interests in these fields. This course is separated into three parts and each part will be taught by an experienced researcher in this field.

Part I. An introduction to the Donaldson theory

Lecturer: Prof. Bohui Chen, Sichuan University



Bio: Bohui Chen got his Ph.D. from the University of Wisconsin-Madison in 2001, and was the Moore Instructor of MIT during 2001-2004. He became a full professor of Sichuan University and the vice dean of the School of Mathematics since 2004. His main research interests include the geometric and topological invariants from various types of moduli spaces.

Lecture time: March 18, 25, April 1, 8.

20:00-21:30 Beijing time (15:00 - 16:30 Moscow time)

Join Zoom Meeting:

<https://zoom.us/j/84169649270?pwd=d3l5dzBqRXBYUHltV3JSajM5NkZvZz09>

Meeting ID: 841 6964 9270

Passcode: 871999

Syllabus:

The course consists of four lectures, which is designed to be a brief introduction to the Donaldson theory on smooth 4-manifolds. It will cover the following topics:

- (1) Moduli spaces of $SU(2)$ -instantons and the Donaldson theorem;
- (2) A brief tour to the Donaldson invariants;
- (3) Gluing techniques and its applications;
- (4) The Floer's homology groups.

References:

- [1]. The geometry of four-manifolds, S.K Donaldson and P.B. Kronheimer, Oxford University Press,1990.
- [2]. Floer homology groups in Yang-Mills theory, S. Donaldson, Cambridge University Press,2004.

Part II. Higgs bundles and related topics

Lecturer: Dr. Qiongling Li, Chern Institute of Mathematics, Nankai University



Bio: Qiongling Li got her Ph.D. from Rice University in 2014. She is currently a research fellow at Chern Institute of Mathematics, Nankai University. Her main research fields are Higgs bundles, harmonic maps, and higher Teichmuller theory. Her recent works have been focused on understanding the non-abelian Hodge correspondence over Riemann surfaces.

Lecture time: April 12, 15, 19, 22, 26;
May 3, 10, 17, 24, 27, 31,
June 3.

20:00-21:30 Beijing time (15:00 - 16:30 Moscow time)

Join Zoom Meeting:

<https://zoom.us/j/86277758883?pwd=Y2FOc2YyWDNJTjc3NWEzcVRGaXlGdz09>

Meeting ID: 862 7775 8883

Passcode: 837467

Syllabus:

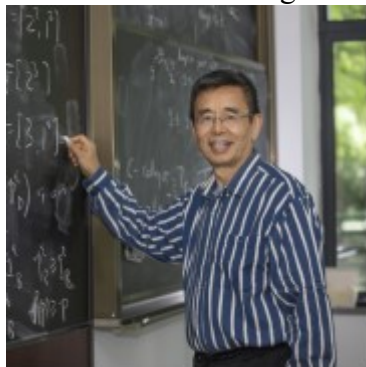
Higgs bundles over a complex manifold is a natural generalization of holomorphic vector bundles and variation of Hodge structures. The main goal in this short course is to introduce Higgs bundles and associated research topics. Firstly, the non-abelian Hodge correspondence gives a homeomorphism between the representation variety of the surface group into a noncompact Lie group and the moduli space of Higgs bundles. In this way, Higgs bundles plays an important role in the study of higher Teichmuller theory as the generalization of classical Teichmuller theory into higher

rank Lie groups. Secondly, the Hitchin fibration on the moduli space of Higgs bundles shows it is a classical integrable system which links with geometric Langlands correspondence.

This short course consists of two parts. In the first part, I will introduce Higgs bundles, moduli spaces, Hitchin integrable system, spectral curve, non-abelian Hodge correspondence, higher Teichmüller theory, etc. I will show some lower rank examples to link Higgs bundles with explicit geometry. In the second part, I will explain current developments on several selected research topics such as Hitchin WKB problem, harmonic maps for Hitchin representations, algebraic structures of the nilpotent cone, and the Hitchin-Kobayashi correspondence over non-compact surfaces.

Part III. An introduction to geometrical Langlands program

Lecturer: Prof. Yongbin Ruan, IAS of Zhejiang University



Bio: Yongbin Ruan is currently a professor at IAS of Zhejiang University and is a world well-known mathematician. He was selected as an Academician of the Chinese Academy of Sciences since 2021. He works in the field of symplectic geometry and mathematical physics. He was the founder of many famous theories in mathematics, like (orbifold) Gromov-Witten theory, relative Gromov-Witten theory, Chen-Ruan cohomology and Fan-Jarvis-Ruan-Witten theory. He has made great contributions to the resolution of many famous conjectures, like Arnold conjecture, the generalized Witten conjecture and LG/CY correspondence conjectures.

Lecture time: twice in June (TBA)