Set of problems 3

14.10.2022

1. Prove that the sequence of random variables $\xi_0, \xi_1, \xi_2, \dots$ in the Galton-Watson model forms a Markov chain. Compute its transition probabilities in the case $\mathbb{P}(\eta_n^i = 2) = p$ and $\mathbb{P}(\eta_n^i = 0) = 1 - p$, where $0 < p < 1$ (you can think of a bacteria that divides by two with probability p or dies without leaving offspring with probability $1 - p$).

2. There is a number of bacterias. Every millisecond each bacteria, independently from the other bacterias, either divides by 2 with proba 0,2, or dies with proba 0,1 or does nothing with proba 0,7. Assume that we start from one bacteria.

   (1) Find a probability that one day all bacterias are dead.

   (2) Find mathematical expectation and variance of the number of bacterias at time equal to n milliseconds.

   (3) Solve the above problems assuming that at the initial moment of time we have not one but $m \in \mathbb{N}$ bacterias.

   Hint: How the probability of extinction is related with the generating functions?

3. Nikolai, a candidate for Deputy of the district council of the village Markovka, does not have enough money for his election campaign. He announced a crowdfunding company in the village and collected 300 rubles but to start the campaign he needs 800 rubles. The local rich man Ermil, who is the owner of the beer shop in the village, suggested to Nikolai to play the following game. If Nikolai bets A rubles, he wins A rubles with probability 0.4 and loses A rubles with probability 0.6. Of course, Nikolai cannot bet more money than he has.

   Draw the corresponding to this game random walk. Find a probability that Nikolai will be able to start the election campaign before he will lose all money he got by the crowdfunding company if

   1) accordingly to the advise of his mom, he bets exactly 100 rubles each time

   2) accordingly to the advise of his dad, each time he bets the largest possible amount (but such that in the case of winning he will have no more than the required 800 rubles).

   Which strategy leads to higher probability to start the election campaign?

   Hint: Consider the probabilities $\phi(x)$ that Nikolay will make 800 rubles before losing all the money, given that he has x rubles at the beginning, for various x. How are they related?