Course Title (in English)	Quiver Representations and Quiver Varieties
Course Title (in Russian)	Представления колчанов и колчанные многообразия
Lead Instructor	Feygin, Evgeny

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Course Description

The theory of quivers is one of the central topics in various fields of modern mathematics and mathematical physics, such as algebraic geometry, representation theory, combinatorics, quantum field theory, integrable systems. The theory has lots of beautiful and deep theorems and is very popular due to a huge number of applications, including McKay correspondence, instantons and ADHM construction, geometric realization of the Kac-Moody Lie algebras. Many of the recent results and applications of the theory of quivers are based on the quiver verieties, introduced by Hiraku Nakajima 20 years ago. The course will cover the basic material on the structure theory of quivers and their representations, such as path algebras, Gabriel's theorem, Hall algebras, preprojective algebras and Auslander-Reiten quivers. Based on the general theory of quiver representations we will discuss the definition of the Nakajima quiver varieties and several explicit examples and applications. The course is aimed at the graduate students or advanced bachelor students. The basic knowledge of algebraic geometry, differential geometry, and the theory of Lie groups and Lie algebras is expected.

Course Prerequisites / Recommendations	Basic knowledge of algebraic geometry, differential geometry, and the theory of Lie groups and Lie algebras.

Аннотация

Теория представлений колчанов играет важнейшую роль в ряде областей современной математики и математической физики, таких как алгебраическая геометрия, теория представлений, комбинаторика, квантовая теория поля, интегрируемые системы. Теория содержит большое число глубоких и красивых конструкций и теорем и получила широкое распространение благодаря большому числу приложений, таких как соответствие Маккея, инстантоны и конструкция АДХМ, геометрическая реализация алгебр Каца-Муди. Большое число современных результатов и приложений теории колчанов основаны на конструкции колчанных многообразий, предложенной Хираку Накаджимой около 20 лет назад. В нашем курсе мы обсудим основы структурной теории и теории представлений колчанов. В частности, мы изучим алгебры путей, докажем теорема Габриэля, введём понятия алгебр Холла и препроективных алгебр, изучим колчаны Ауслaндера-Рейтен. Используя общие сведения из теории колчанов, мы определим колчанные многообразия Накаджимы и изучим их основные свойства и приложения. Курс рассчитан на магистров, аспирантов и сильных студентов старших курсов бакалавриата. Предполагаются, что слушатели владеют базовыми понятиями алгебраической и дифференциальной геометрии, а также теории групп и алгебр Ли.

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Basic theory: the Dynkin case	Basic notions and definitions for the theory of Dynkin quivers.	1	3	2
Geometry of orbits, Gabriel's theorem	Descripition of the group action, a link to the theory of simple Lie algebras,	1	3	2
Hall algebras, preprojective algebras	Definitions, main properties and applications of Hall algebras and preprojective algebras.	1	3	2
Auslander-Reiten quivers	Basics on the Auslander-Reiten theory.	1	3	2
Tame and wild quivers	Main properties of the tame and wild quivers: structure theory and representation theory.	1	3	2
Hamiltonian reduction and geometric invariant theory	The first step towards the construction of the Nakajima quiver varieties.	1	3	2
Nakajima quiver varieties	Definition of the Nakajima quiver varieties.	1	3	2
Hilbert schemes	Hilbert schemes in the context of Nakajima quiver varieties.	1	3	2
Applications	Various applications of the theory of Nakajima quiver varieties. Realization of Kac-Moody Lie a;gebras.	1	3	2

Assignment Type	Assignment Summary
Homework Assignments	Homeworks consisting of problems on the topics covered within the course.
Test/Quiz	Tests on the subjects to be discussed within the course.
Midterm Exam	Midterm exam consists of problems on the topics covered within the first half of the course.
Final Exam	Final exam consists of problems on the topics covered within the course.

Type of Assessment	Graded

Grade Structure

Activity Type	Activity weight, %
Final Exam	40
Homework Assignments	25
Midterm Exam	20
Test/Quiz	15

A:	85
B:	75
C:	65
D:	55
E:	45
F:	0
Attendance Requirements	Optional with Exceptions

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1 Term 2
Course Delivery Frequency	Every two years

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Course Tags	Math

Required Textbooks	ISBN-13 (or ISBN-10)
Alexander Kirillov Jr., Quiver Representations and Quiver Varieties, 2016	9781470423070

Papers	DOI or URL
V.Ginzburg, Lectures on Nakajima's quiver varieties, Geometric methods in representation theory. I, Semin. Congr., vol. 24, Soc. Math. France, Paris, 2012, pp. 145–219.	https://arxiv.org/abs/0905.0686

Knowledge
Basics of the theory of quivers. Basics on the Nakajima quiver varieties.

Skill
An ability to work with various objects of algebraic, geometric and combinatorial origin

Experience
Experience in apliccation of modern mathematical theory in mathematcal physics.

Select Assignment 1 Type	Homework Assignments
Input Example(s) of Assignment 1 (preferable)	Let $X,Yin End(V)$ be such that $rk[X,Y] = 1$. Show that X and Y are simultaneously triangularizable. A K-algebraAis called connected if 0 and 1are the only idempotents that lie in the center of A. Show that the path algebra KQ is connectedif and only if the quiver Q is connected. Describe all (finite dimensional) indecomposable representations of the equiotriented type A quiver (up to isomorphism) and all possible morphisms between them.
Assessment Criteria for Assignment 1	Each problem is evaluated out of 4 points. 4 points means excellent; 3 points - minor corretions needed; 2 points - the main idea is described, but important details are missing; 1 point - only a vague idea is given.
Select Assignment 2 Type	Test/Quiz
Input Example(s) of Assignment 2 (preferable)	Describe the path algebra of the one loop quiver. Construct infinitely many indecomposable representations of the one-loop quiver. Describe the path algebra of the Kronecker quiver.

Assessment Criteria for Assignment 2	Each problem is evaluated out of 4 points. 4 points means excellent; 3 points - minor corretions needed; 2 points - the main idea is described, but important details are missing; 1 point - only a vague idea is given.
Select Assignment 3 Type	Final Exam
Input Example(s) of Assignment 3 (preferable)	Let Q be the type A_1 quiver (one vertex, no arrows). Compute the dimension of the preimage of zero with respect to the moment map. Let Q be the quiver with one vertex and one edge. Compute the dimension of the preimage of zero with respect to the moment map.
Assessment Criteria for Assignment 3	Each problem is evaluated out of 4 points. 4 points means excellent; 3 points - minor corretions needed; 2 points - the main idea is described, but important details are missing; 1 point - only a vague idea is given.
Select Assignment 4 Type	Midterm Exam
Input Example(s) of Assignment 4 (preferable)	Prove that for any root d of a Dynkin quiver there exists an indecomposable representation of dimension d. Assume that a quiver Q has at least one loop. Prove that Q has infinite number of pairwise non isomorphic indecomposable representations.
Assessment Criteria for Assignment 4	Each problem is evaluated out of 4 points. 4 points means excellent; 3 points - minor corretions needed; 2 points - the main idea is described, but important details are missing; 1 point - only a vague idea is given.