Set of problems 9

09.12.2022

1. Prove the second item of the Perron-Frobenius theorem.

2. Under the assumptions of the Perron-Frobenius theorem show that if μ is an eigenvalue of the matrix A then either $\mu = \lambda$ or $|\mu| < \lambda$.

3. Under the assumptions of the Peffon-Frobenius theorem prove that the left and right eigenspaces associated with the eigenvalue λ are one dimensional.

4. Prove that if a state i has period one then there is $n_i \geq 1$ such that $p_{ii}^{(n)} > 0$ for every $n \geq n_i$.

   Hint: Use that for any $m, k \in \mathbb{N}$ satisfying $gcd(m, k) = 1$ there are $a, b \in \mathbb{N}$ such that $am - bk = 1$ (prove this!).

5. An irreducible Markov chain is called *aperiodic* if its period equals one. Using the previous exercise prove that: **an irreducible aperiodic Markov chain** (with finite number of states) has mixing transition probability matrix and consequently **is exponentially ergodic.**