Task 3. Automorphisms. Complex dynamics

April 8, 2024

Deadline: April 30

Problem 1. Find a biholomorphic automorphism of the unit ball in $\mathbb{C}^2$ that sends the origin to the point
a) $(\frac{1}{4}, 0)$;
b) $(\frac{1}{2}, \frac{1}{2})$.

Problem 2. (a), b)). Find similar automorphisms of the unit polydisk in $\mathbb{C}^2$.

Problem 3. Find which ones from the next collections of complex numbers are resonant:
a) $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$;
b) $(\frac{1}{3}, \frac{1}{7}, \frac{1}{8})$;
c) $(-\frac{1}{2}, \frac{1}{8}, \frac{1}{3})$.

Problem 4. Prove that the basin of attraction of the origin of the map $(z, w) \mapsto (\frac{1}{2}z, \frac{1}{3}w + z^2)$ is all of $\mathbb{C}^2$.

Problem 5. Find the lower nonlinear term of k-th iterate of the germ of conformal mapping

$$f(z) = az + z^m + O(z^{m+1}),\ m \geq 2,\ a \in \mathbb{C}^*.$$

Problem 6. Provide an example of *non-linear* polynomial involution $f : \mathbb{C}^2 \to \mathbb{C}^2$, $f^2 = Id$, fixing the origin, with $f'(0) = -Id$.

Hint. Construct the above $f = (f_1, f_2)$ with $deg f_j \leq 2$.

Problem 7. Let $U \subset \mathbb{C}^2$ be a Fatou-Bieberbach domain: the basin of a non-resonant attractive fixed point of a polynomial automorphism that does not coincide with all of $\mathbb{C}^2$.

1) Prove that the intersection with U of every complex line is simply connected. *Hint.* Use Maximum Principle.

2)Using the result of the above problem prove that the unit disk can be realized as a one-dimensional submanifold in $\mathbb{C}^2$.