

# Analysis of several complex variables. Program of final exam. May 21, 2024

May 15, 2024

1. Holomorphic functions of several complex variables. Cauchy–Riemann equations. Multidimensional Cauchy integral formula for separately holomorphic functions.
2. Hartogs’ Theorem on equivalence of holomorphicity and separate holomorphicity (without proof). Proof under continuity assumption: Osgood Lemma.
3. Theorem on decomposition in converging Taylor series. Abel’s Lemma on convergence of power series.
4. Convergence domain of power series: its main properties. Description of its boundary (multidimensional analogue of Cauchy-Hadamard formula for convergence radius).
5.  $C^\infty$ -smoothness of holomorphic function. Equivalence of definition of holomorphic function as a converging power series and formula relating its coefficients with derivatives.
6. Uniqueness of analytic extension. Openness, Maximum Principle, Liouville Theorem.
7. Hartogs’ Theorem on extension of holomorphic function from a Hartogs figure to the ambient polydisk. Corollary: erasing compact singularities in a polydisk.
8. Holomorphic Implicit Function and Constant Rank Theorems.
9. Complex manifolds and holomorphic functions on them. Complex submanifolds. Erasing codimension two singularities (and codimension one singularities for bounded functions).
10. Analytic sets, regular and singular parts: definition and main properties. Closeness under finite unions and intersections.
11. Nowhere density of proper analytic subset in a connected manifold  $M$ . Density of regular part.

12. Dimension: definition. Analyticity theorem for singular part (without proof). Irreducibility criterion for analytic subsets (without proof). Proof of irreducibility of analytic subset with connected regular part.
13. Analyticity of preimage of analytic set. Remmert Proper Mapping Theorem (without proof). Theorem on decomposition into irreducible components (without proof).
14. Weierstrass polydisk and polynomials. Weierstrass Preparatory Theorem.
15. Local ring. Theorem on its factoriality.
16. Zero locus of function as a covering over the complement to an analytic subset. Geometric factorization into irreducible factors and irreducibility criterion.
17. Irreducibility criterion for germs of holomorphic functions in two variables in terms of zero locus and multiplicities.
18. Newton diagrams. Additivity under multiplication of functions. Theorem on relation between edges and irreducible factors.
19. Generalized Maximum Principle and Schwarz Lemma.
20. Classification of biholomorphic automorphisms of polydisk.
21. Classification of biholomorphic automorphisms of ball.
22. Introduction to complex dynamics. Linearization theorem in dimension one.
23. Linearization theorem for germs of biholomorphic maps in dimension two.
24. Polynomial automorphisms of  $\mathbb{C}^2$ . Biholomorphicity of basin of attraction and  $\mathbb{C}^2$ . Fatou–Bieberbach domains.
25. Domains of holomorphy and holomorphic convexity. Oka’s theorem on equivalence of these notions. Proof of the first part: holomorphic convexity implies domain of holomorphy.
26. Cartan – Thullen Theorem. Proof of the second part of Oka’s theorem: domain of holomorphy implies holomorphic convexity.
27. Holomorphic non-extendability. Behnke–Sommer Continuity Principle. Oka’s theorems (without proof) on relation between Levi convexity, being domain of holomorphy and holomorphic non-extendability. Domain of holomorphy implies Levi convexity (with proof).
28. Levi form. Levi – Krzoska Theorem on necessary and sufficient Levi convexity conditions for domains with  $C^2$ -smooth boundary.
29. Pluri(sub)harmonic functions. Levi convexity of their sublevel sets.
30. Stein manifolds: definition and embedding theorem (without proof). Dolbeault cohomology.  $\bar{\partial}$ -Poincaré Lemma in one variable.

31. Proof of  $\bar{\partial}$ -Poincaré Lemma in many variables. Statement of its generalization to Stein manifolds (without proof).

32. Multiplicative Cousin problem. Global defining function for a hypersurface: relation between its existence and solution of multiplicative problem.

33. Solution of holomorphic additive Cousin problem for polydisk.