

## Homework #1.

1. Genetic model. For a relatively realistic example of Markov chains we consider an idealized genetic model introduced by S. Wright. The basic idea of the model is to investigate the fluctuation of gene frequency in a population under the influence of selection. Let us in our model disregard effects as mutation or selective forces for simplicity. The population size is assumed to be fixed throughout every step in our model. Our population shall contain  $2N$  individuals either having a type-a genetic characteristic or a type-A characteristic. The next generation is determined by  $2N$  independent binomial trials. This means that one individual of the parent generation is randomly chosen and the successor is from the same genetic type.

Let us assume that the population contains  $j$  type-a individuals. All the other individuals are of type-A.

Question 1. Which probabilistic concept drives this model? What is the probability that a randomly chosen individual of the parents generation is of type-a or of type-A respectively?

The question arising in this genetic model is whether one or the other genetic type can die out at some time and if what is the probability for this. To answer this question we need an appropriate model for this problem.

Question 2. Argue why it makes sense to use Markov chains for modelling this problem.

Let us assume for simplicity that our population contains four individuals that is we have  $N = 2$ .

Question 3. (1) What are the possible states of the Markov chain?

(2) Are there absorbing states in this model? If yes, which are the absorbing states?

The next step to fully describe the Markov chain are the transition probabilities.

Question 4. Give the transition matrix in this model.

Question 5. Will always one genetic type die out? Or more exactly: Do we observe with a probability of 1 either of the states 0 or 4?

2. A coin is tossed three times. What is the probability that exactly two heads occur, given that

(a) the first outcome was a head?

(b) the first outcome was a tail?

(c) the first two outcomes were heads?

(d) the first two outcomes were tails?

(e) the first outcome was a head and the third outcome was a head?