What to read?

1. Any classical book in probability theory (Shiryaev, Sinai - Koralov, Feller etc) contains a chapter dedicated to Markov chains and can be used for our course. The only issue is that all these books are real probability textbooks so assume that the reader is ready to deal with sigma algebras etc.
2. Markov Chains by J. R. Norris is a good textbook that contains not only discrete but also continuous Markov chains description.
3. Markov Chains and Mixing times, D. Levin - Yu. Peres is a very good book that covers much more than we will be able to study but is very useful: https://pages.uoregon.edu/dlevin/MARKOV/mcmt2e.pdf
4. Ю. Сухов. М. Кельберт Марковские цепи как отправная точка случайных процессов и их приложений (том 2) - интересная книжка с большим количеством исторических сведений и примеров

What to watch?
1. Yuval Peres on Markov chains: https://www.youtube.com/watch?v=wepF7TmFNWA