

## Homework 2

15 сентября 2024 г.

**Problem 1.** In the Dark and Crazy Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale.

- Find the probability that the grandson of a man from Harvard went to Harvard.
- Modify the above by assuming that the son of a Harvard man always went to Harvard. Again, find the probability that the grandson of a man from Harvard went to Harvard.

**Problem 2.** Start a random walk on a vertex of a cube, with equal probability going along the three edges that you can see (to another vertex). what is the expected number of steps to reach the opposite vertex that you start with?

**Problem 3.** Let there be given two individuals with genotype  $Aa$ . Consider a sequence of trials, each consisting of drawing two individuals at random from the offsprings of the previous generation. Let  $X_n$  state the genotypes for the individuals drawn in the  $n$ 'th trial,  $n \in \mathbb{N}$ . Thus  $X_n$  can take 6 different values:

$$E_1 = \{AA, AA\};$$

$$E_2 = \{AA, Aa\};$$

$$E_3 = \{Aa, Aa\};$$

$$E_4 = \{Aa, aa\};$$

$$E_5 = \{aa, aa\};$$

$$E_6 = \{AA, aa\}.$$

Assume that the probability for A respectively a is  $\frac{1}{2}$ . Find the transition matrix and classify the states for this homogeneous Markov chain. Determine the absorption probabilities  $\alpha_j(C)$  for all transient states  $j$  and  $C = \{E_1\}$  respectively  $C = \{E_5\}$ .