Homework 4

19 октября 2024 г.

Problem 1. Below a series of transition matrices for homogeneous Markov chains is given. Draw (or sketch) the transition graphs and examine whether the chains are irreducible. Classify the states.

1. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

2. $\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Problem 2. Consider the Markov chain with transition matrix $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{3}{4} & 0 & \frac{1}{4} \\ 0 & 1 & 0 \end{pmatrix}$ Find the matrix which is the limit of P^n as $n \to \infty$.

Problem 3. Show that a Markov chain with transition matrix $P = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 \end{pmatrix}$ has more than one stationary distributions. Find the matrix that P^n converges to, as $n \to \infty$, and verify that it is not a matrix all of whose rows are the same.