List of problems 2

November 14th, 2024

1 Instructions

The date of issue: November 14th, 2024.
The deadline (non-extendable): November 22nd, 2024 (23:59).
Please send scanned version of your solution **in one file** to Alexey Kobzev:
akobzev@hse.ru
Once the result will be published, those who solved correctly **at least two problems** will be given 10 minutes appointment (in person or online) and asked to explain the solution of one of the solved problem (on my choice). In case of successful presentation you will be given **1 point** (for the whole list).

2 Problems

Problem 1. Toss a fair die repeatedly. Let S_n denote the total of the outcomes through the nth toss. Show that there is a limiting value for the proportion of the first n values of S_n that are divisible by 7, and compute the value for this limit.

Problem 2. In unprofitable times corporations sometimes suspend dividend payments. Suppose that after a dividend has been paid the next one will be paid with probability 0.9, while after a dividend is suspended the next one will be suspended with probability 0.6. In the long run what is the fraction of dividends that will be paid?

Problem 3. A transition matrix P is said to be doubly stochastic if $\sum_{i=0}^{M} P_{ij} = 1$ for all states $j = 0, 1, ..., M$. Show that if such a Markov chain is ergodic, with $(\pi_0, \cdots, \pi_M)$ the stationary row vector, then $\pi_j = \frac{1}{(M+1)}, j = 0, 1, ..., M$.

Problem 4. For a classical random walk on $\mathbb{Z}$ with parameter p use the strong law of large numbers to give another proof that the Markov chain is transient if the parameter $p \neq \frac{1}{2}$.