List of problems 1

6 октября 2024 г.

1 Instructions

The date of issue: October 3rd, 2024.
The deadline (non-extendable): October 14th, 2024 (23:59).
Please send scanned version of your solution **in one file** to Alexey Kobzev: akobzev@hse.ru
Once the result will be published, those who solved correctly **at least two problems** will be given 10 minutes appointment (in person or online) and asked to explain the solution of one of the solved problem (on my choice). In case of successful presentation you will be given **1 point** (for the whole list).

2 Important definitions

- two events A and B are independent if $P(A \cap B) = P(A)P(B)$. Two real valued random variables X and Y are independent if for all x and y the events $\{X \leq x\}$ and $\{Y \leq y\}$ are independent.
- A Markov chain is called *homogeneous* if and only if the transition probabilities are independent of the time t.

3 Problems

Problem 1. Let $Y_0, Y_1, \cdots$ be a sequence of independent identically distributed random variables on $\mathbb{Z}$ such that

$$P(Y_n = 1) = P(Y_n = -1) = \frac{1}{2}, n = 0, 1, 2, \cdots .$$

Consider the stochastic process $\{X_n\} = \frac{Y_n + Y_{n+1}}{2}$. Check if $\{X_n\}$ is a homogeneous Markov chain.

Problem 2. The student Hardworker is trying to pass an exam. With an equal probability Hardworker can get any integer grade from 0 to 10. Find an expected number of the trials before Hardworker at 3 consequent exams gets exactly the same mark. Try to use Markov chains to get the answer.
Problem 3. Assume that the sequence of random variables $\sigma_1, \cdots, \sigma_n$ forms a homogeneous Markov chain with the transition probabilities p_{ij}. Is it true that $\sigma_n, \cdots, \sigma_1$ also forms a homogeneous Markov chain? if not, find a condition on the initial distribution that will guarantee that the reverse sequence is indeed a homogeneous Markov chain.
Problem 4. Let us consider the simple random walk on $\mathbb{Z}^2$. Transition probabilities are given by the formula $p_ij = \frac{1}{4}$ if $|i-j| = 1$ and $p_{ij} = 0$ otherwise. Check is the corresponding Markov chain is transient or recurrent.