

## Special functions. Problems for seminar 3

1. a) Compute Dirichlet integral

$$\int_{x_1 > 0, \dots, x_n > 0, \sum x_i < 1} x_1^{\alpha_1 - 1} \dots x_n^{\alpha_n - 1} dx_1 \dots dx_n, \quad \operatorname{Re} \alpha_i > 0$$

- b) Compute the volume of ellipsoide

$$\sum_{i=1}^n \left( \frac{x_i}{a_i} \right)^{p_i} \leq 1$$

2. Show that for  $\operatorname{Re} z > 0$

$$\frac{d^2 \log \Gamma(z)}{dz^2} = \int_0^\infty \frac{te^{tz}}{1 - e^t} dt$$

3. Find first three terms of asymptotical expansion over big  $n$  of  $n$ -th positive root of the equation

$$x \sin x = 1$$