

Special functions. Problems for seminar 6

1. Generalize Riemann functional equation to Hurwitz ζ function:

$$\zeta(x, s) = \frac{2\Gamma(1-s)}{(2\pi)^{1-s}} \left\{ \sin \frac{s\pi}{2} \sum_{n=1}^{\infty} \frac{\cos 2\pi nx}{n^{1-s}} + \cos \frac{s\pi}{2} \sum_{n=1}^{\infty} \frac{\sin 2\pi nx}{n^{1-s}} \right\}, \quad \operatorname{Re} s < 0$$

2. Show that the function $f(z) = \operatorname{ctg} z$ is a meromorphic function, bounded by a constant on a system of contours encircling poles (e.g. circles

$$C_n : |z| = \left(n + \frac{1}{2}\right) \pi$$

or proper rectangles). Decompose $f(z)$ into a convergent series of elementary fractions¹.

¹better to work with $\operatorname{ctg} z - \frac{1}{z}$