Special functions. Problems for seminar 8

1. Using integral presentation for Gauss hypergeometric function, derive Pfaff and Euler identities

$$F_{2,1}(a,b;c;z) = (1-z)^{-a} F_{2,1}\left(a,c-b;c;\frac{z}{1-z}\right),$$

$$F_{2,1}(a,b;c;z) = (1-z)^{c-a-b} F_{2,1}\left(c-a,c-b;c;z\right),$$

- 2. a) The function $(1-x)^{-a}$ coincides with hypergeometric function $F_{1,0}(c;x)$ so that Euler integral presentation of Gauss hypergeometric function $F_{2,1}(a,b;c;z)$ can be regarded as recursive integral presentation of $F_{2,1}$ hypergeometric function via $F_{1,0}$ hypergeometric function. Write down analogous integral formula presenting $F_{3,2}(a,b,c;d,e;x)$ via $F_{2,1}$ hypergeometric function.
 - b) Derive as a corollary the double integral, presenting $F_{3,2}(a,b,c;d,e;x)$.