Special functions. Problems for seminar 10

1. Barnes theorem says that for $\arg(-x)| < \pi - \delta$

$$F_{2,1}(a,b;c;x) = \frac{\Gamma(c)}{2\pi i \Gamma(a)\Gamma(b)} \int_{-i\infty}^{i\infty} \frac{\Gamma(a+t)\Gamma(b+t)}{\Gamma(c+t)} \Gamma(-t)(-x)^t dt. \qquad (1)$$

   We can try to compute the same integal closing the contour to the left half plane. What the result we will get in this way?

2. Check directly that Barnes integral (1) satisfies Gauss hypergeometric equation. [1]

3. Barnes theorem can be interpreted as the inversion formula for Mellin transform of $F(a,b;c;-x)$. Compute directly Mellin transform of $F(a,b;c;-x)$ without using Barnes theorem.

[1] It is more convenient here to use the equation in a form $\left(x\frac{d}{dx}+a\right)\left(x\frac{d}{dx}+b\right)y = \frac{d}{dx}\left(x\frac{d}{dx}+c-1\right)y$