

Special functions. Problems for seminar 11

1. Prove the Barnes first lemma

$$\frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \Gamma(a+s)\Gamma(b+s)\Gamma(c-s)\Gamma(d-s)ds = \frac{\Gamma(a+c)\Gamma(a+d)\Gamma(b+c)\Gamma(b+d)}{\Gamma(a+b+c+d)},$$

$\operatorname{Re} a, b, c, d > 0$ by residue calculations.

2. Calculate the integral

$$\int_{-i\infty}^{+i\infty} \frac{\Gamma(a+s)\Gamma(c-s)}{\Gamma(b+s)\Gamma(d-s)}, \quad \operatorname{Re} a, b, c, d > 0$$

Explain (or prove) the result by means of Plancherel formula for Mellin transform.