Special functions. Problems for seminar 13

1. Rewrite Heine transformation formula

$$\varphi_{21}(q^a, q^b; q^c; z|q) = \frac{(q^b;q)_\infty (q^a z;q)_\infty}{(q^c;q)_\infty (z;q)_\infty} \varphi_{21}(q^{c-b}, z; q^a z; q^b|q)$$

   as a q version of Euler integral identity

$$\int_0^1 x^{a-1}(1-x)_q^{b-1} d_q x = \frac{\Gamma_q(a)\Gamma_q(b)}{\Gamma_q(a+b)}$$

   where

$$(1-x)_q^{b-1} := \frac{(qx;q)_\infty}{(q^b x;q)_\infty}$$

   according to q binomial theorem

2. In $\psi_{1,1}$ Ramanujan summation formula

$$\sum_{n=-\infty}^{\infty} \frac{(a;q)_n}{(b;q)_n} x^n = \frac{(ax, q/ax, b/a, q|q)_\infty}{(x, b, q/a, b/ax|q)_\infty}$$

   substitue first $b = 0$, then replace x by x/a and then tend a to infinity. At the end there should be a proof of Jacoby triple product identity for θ_1 function:

$$\theta(x) := (q;q)_\infty (x;q)_\infty (q/x;q)_\infty = \sum_{n=-\infty}^{\infty} (-1)^n q^{n(n-1)/2} x^n$$