

## Special functions. Problems for seminar 14

- 1.
2. Multiple Hurwitz zeta function  $\zeta(s, z|\omega_1, \dots, \omega_r)$  is defined by the integral over Hankel contour

$$\zeta(s, z|\omega_1, \dots, \omega_r) = -\Gamma(1-s) \int_C \frac{e^{-zt}(-t)^{s-1}}{\prod_{k=1}^r (1 - e^{-\omega_k t})} \frac{dt}{2\pi i}$$

Calculate values of  $\zeta(s, z|\omega_1, \dots, \omega_r)$  in negative integer points. Express them via multiple Bernoulli polynomials

3. Let  $f(t)$  be a function, holomorphic in a vicinity of real line and fast decaying when  $t \rightarrow \infty$ . Prove that

$$\begin{aligned} \text{a)} \quad & \int_C f(t) \log(-t) \frac{dt}{2\pi i t} = - \int_0^\infty f'(t) \log t dt, \\ \text{b)}^* \quad & \int_C f(t) \log(-t) \frac{dt}{2\pi i t^2} = f'(0) - \int_0^\infty f''(t) \log t dt, \end{aligned}$$