Introduction to KAM theory. Exam program

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1) Linearization theorem for germs of conformal maps

$$f(z) = \lambda z(1 + o(1)), \quad |\lambda| \neq 0, 1.$$

2) Diophantine numbers. Prove that their set is of full measure. Statements of Siegel Linearization Theorem and Bruno and Yoccoz Theorems.

3) Statement of the Main Lemma for Siegel Theorem. Proof of Siegel Theorem modulo the Main Lemma.

4) Cauchy bounds in one complex variable. First statement of the Main Lemma for Siegel Theorem: solution of homological equation and its bound.

5) Proof of the second statement of the Main Lemma for Siegel Theorem: quadratic error bound, i.e., upper bound for $f_1(z) - \lambda z$.

6) Rotation number. Definition, statement of the theorem on its well-definedness. Elementary properties. Proofs of its continuity and monotonicity. Statements of two Denjoy Theorems and Herman Conjugation Theorem.

7) Analytic circle diffeomorphisms close to rotations. Statement of Kolmogorov–Arnold analytic conjugacy theorem. Statement of the Main Lemma and proof of the conjugacy theorem modulo the Main Lemma.

8) The Main Lemma for Kolmogorov–Arnold analytic conjugacy theorem for circle diffeomorphisms close to rotations. Proof of its first part: solution of homological equation and its bound.

9) The Main Lemma for Kolmogorov–Arnold analytic conjugacy theorem for circle diffeomorphisms close to rotations. Proof of its second part: quadratic error bound, i.e., the bound of distance of the $f_1(\phi)$ to a rotation.

10) Symplectic manifolds. Examples. Hamiltonian vector fields and their basic conservation laws. Cartan formula for Lie derivative of a form.

11) Poisson bracket. Jacobi identity. Lie algebra of functions and its isomorphism with the Lie algebra of Hamiltonian vector fields.

12) Canonical coordinates. Darboux theorem on their local existence.

13) Symplectomorphisms. Basic examples, including natural symplectomorphisms of cotangent bundles. Generating functions and local space of symplectomorphisms.

14) Integrable systems. Arnold–Liouville Theorem. Proof of its first part: on tori and the flow on them.

15) Arnold–Liouville Theorem. Proof of its second part: construction of action-angle coordinates.

16) Diophantine vectors and their genericity in measure sense for $\gamma > n-1$. Statements of the two Kolmogorov-Arnold invariant tori theorems on persistence of invariant tori with Diophantine frequencies. Deduction of the theorem on persistence of many tori, from the theorem on one torus.

17) The Main Lemma for Kolmogorov–Arnold theorem on persistence of individual Diophantine invariant torus. Proof of the persistence theorem modulo the Main Lemma.

18) Cauchy bounds in many complex variables. The Main Lemma for Kolmogorov–Arnold theorem on persistence of individual Diophantine invariant torus. Proof of its first part: solution of homological equation and its bound.

19) The Main Lemma for Kolmogorov–Arnold theorem on persistence of individual Diophantine invariant torus. Proof of its second part: quadratic error bound, i.e., bound of free in r and linear in r terms in the new Hamiltonian function H_1.