Moduli, dynamics, and integrability

Place: Faculty of Mathematics of the HSE University, Usacheva str. 6, Moscow

Conference program

Monday, 13 July

10:00 – 11:00	Alexandr Veselov, “Matrix variations on Moser’s theme”
11:00 – 11:30	*Coffee break*
11:30 – 12:30	Andrey Mironov, “Integrable Birkhoff Billiards inside Cones”
12:30 – 15:20	*Lunch*
15:20 – 16:20	Guillaume Tahar, “Low dimensional loci in strata of meromorphic differentials”
16:30 – 17:30	Di Yang, “On Legendre-type transformations of a Frobenius manifold”

Tuesday, 14 July

10:00 – 11:00	Andrey Okounkov, “Quantum cohomology of Hilbert scheme of points in the plane and in the space”
11:00 – 11:30	*Coffee break*
11:30 – 12:30	Maxim Kazarian, “Relations on kappa-classes and spin Gromov–Witten theory of CP^1”
12:30 – 13:50	*Lunch*
13:50 – 14:50	Vladlen Timorin, “Conjugacy invariants of polygon exchange transformations”
14:50 – 15:20	*Coffee break*
15:20 – 16:20	Mauro Artigiani, “An Invitation to Interval Translation Mappings: Renormalization, Weak Mixing, and Exceptions”
16:30 – 17:10	Alexey Kobzev, “Ergodic properties of interval exchange transformations”

Wednesday, 15 July

10:00 – 11:00	Xin Wang, “Universal constraints for Hodge integrals with target varieties and their applications”
11:00 – 11:30	*Coffee break*
11:30 – 12:30	Boris Bychkov, “KP integrability in topological recursion”
12:30 – 13:50	*Lunch*
13:50 – 14:50	Petr Dunin-Barkowski, “A new ELSV-type formula via logarithmic spectral curve topological recursion”
14:50 – 15:20	*Coffee break*
15:20 – 16:00	Alexandr Artemev, “Topological recursion for minimal string theory”

Thursday, 16 July

10:00 – 11:00	Andrey Bogatyrev, “Optimal stability polynomials with third-order tangency condition and theta functions”
11:00 – 11:30	*Coffee break*
11:30 – 12:30	Michael Shapiro, “Poisson–Lie groups and cluster structures”
12:30 – 13:50	*Lunch*
13:50 – 14:50	Vasily Gorbounov, “Generalized electrical Lie algebras and invariants”
14:50 – 15:20	*Coffee break*
15:20 – 16:00	Mikhail Zaitsev, “The q-analog of the Argument Shift Method and the Braided Yangian”
16:10 – 16:50	Alexey Gulyaev, “On the prechromatic weight system on primitive elements”

Friday, 17 July

10:00 – 11:00	Oleg Mokhov, “Realizations of curvature line nets with a given metric in pseudo-Euclidean spaces and integrable hierarchies”
11:00 – 11:30	*Coffee break*
11:30 – 12:30	Alexey Basalaev, “Integrable hierarchies associated to the Hurwitz–Frobenius manifolds”
12:30 – 13:50	*Lunch*
13:50 – 14:50	Yitwah Cheung, “Some recent results on Dani correspondence” (online)
14:50 – 15:20	*Coffee break*
15:20 – 16:00	Ivan Shumilin, “Frobenius manifolds constructed from algebro-geometric data and the integrable hierarchies associated with them”
16:10 – 16:50	Vadim Retinskiy, “A generalization of the matrix tree theorem to the case of pairs of spanning trees”

Abstracts

Alexandr Artemev

Topological recursion for minimal string theory

The long-standing conjecture in physics connects two objects: perturbative amplitudes in certain string theories, obtained by integrating conformal field theory correlation functions over the moduli spaces of curves, and $1/N$-expansion coefficients for observables in two-matrix models, governed by topological recursion. Focusing on the “matrix model” side, we will ex- plain the definition of these objects that was previously developed in the literature and then discuss recent simplified reformulation that incorporates x-y swap transformation of TR (equiv- alence between two definitions was recently proven by Shadrin et al.). Using standard methods, this approach allows to present a general formula for amplitudes in terms of tautological in- tersection numbers; we will provide it and briefly touch on some open questions regarding the interpretation of this formula. The talk is based on arXiv:2506.09222.

Mauro Artigiani

An Invitation to Interval Translation Mappings: Renormalization, Weak Mixing, and Exceptions

Boshernitzan and Kornfeld defined Interval Translation Mappings (ITMs) in 1995 as a natural generalization of Interval Exchange Transformations (IETs). Unlike IETs, ITMs allow for overlapping images. They are split into two categories: finite and infinite type, based on the topology of their attractors. While finite type ITMs essentially reduce to IETs, infinite type ITMs represent a different type of dynamical systems, and their ergodic properties appear to be non-trivial. In this talk, we will survey the dynamical properties of the known examples of infinite type ITMs. By introducing suitable generalizations of the classical Rauzy induction, we construct natural invariant measures to study their typical behavior. We will show that typical Bruin–Troubetzkoy and Bruin ITMs are uniquely ergodic and weakly mixing. Finally, we will present the first known example of an infinite type ITM that is not weakly mixing.

This is based on joint works with A. Avila, S. Ferenczi, C. Fougeron, P. Hubert, and A. Skripchenko.

Alexey Basalaev

Integrable hierarchies associated to the Hurwitz–Frobenius manifolds

It is well known since the constructions of Dubrovin, Zhang, and Buryak that to every semisimple Dubrovin–Frobenius manifold one can associate an integrable system. In this talk, we will focus of the “special” Dubrovin–Frobenius manifolds that have some additional geometric structure – Hurwitz–Frobenius manifolds. We will show that they can be used to construct KP, BKP, CKP, multi-KP, and modifiled KP hierarchies.

Andrey Bogatyrev

Optimal stability polynomials with third-order tangency condition and theta functions

The problem of optimal stability polynomial arises in the synthesis of stabilized explicit multistage Runge-Kutta methods. For the third-order of accuracy method, the problem reduces to finding the moduli of a genus two curve with a marked point, the complexity of which does not depend on the degree of the sought polynomial.

Boris Bychkov

KP integrability in topological recursion

Topological recursion of Chekhov–Eynard–Orantin is a remarkable universal recursive procedure that appears in many problems from enumerative geometry, including combinatorics of maps, random matrices, Gromov–Witten invariants, Hurwitz numbers, Mirzakhani’s hyperbolic volumes of moduli spaces, and knot polynomials. One of the main pieces of initial data for topological recursion is a spectral curve; the recursion then produces the sequence of invariants associated with this curve.

The Kadomtsev–Petviashvili hierarchy is an integrable hierarchy of nonlinear PDEs. In addition to its own significance, it appears in many applications: numerous generating functions arising in combinatorics, mathematical physics, theory of moduli spaces and Gromov–Witten theory are solutions of the KP hierarchy.

In the talk I will explain the appearance of KP tau functions for the topological recursion invariants of genus-zero spectral curves. I will also discuss the construction of KP tau functions for the topological recursion invariants of spectral curves of arbitrary genus. This construc- tion can be viewed as a non-perturbative generalization of Krichever’s construction of KP tau functions associated with algebraic curves.

The talk is based on a series of joint works with A. Alexandrov, P. Dunin-Barkowski, M. Kazarian, and S. Shadrin: arXiv:2309.12176, arXiv:2406.07391, arXiv:2412.18592.

Yitwah Cheung

Some recent results on Dani correspondence

A basic problem in the study of dynamical systems is understanding the statistical long- term behavior of orbits. Ergodic theorems usually only address the generic behavior, but for many special systems that arise naturally from within mathematics, it is possible to get a deeper understanding of the behavior of all orbits. A trivial example is the trajectory of a billiard ball in a square, which is dense and equidistributed if and only if the slope of its initial direction is irrational. There are many examples generalizing this phenomenon, which we call Dani correspondence, where the long-term behavior of orbits are conveniently encoded in the number theoretic properties of the initial parameters. In this talk, I will give a brief overview of some key developments along this theme in the last two decades; in particular, I will mention a generalization of the Khintchine–Levy theorem to higher dimensions (joint with N. Chevallier) and a new result of Y. Wei concerning dichotomy of Hausdorff dimension of nonergodic directions that gives an application to number theory, namely simultaneous Perez Marco condition is independent of choice of norm.

Petr Dunin-Barkowski

A new ELSV-type formula via logarithmic spectral curve topological recursion

It is known that the standard Chekhov–Eynard–Orantin spectral curve topological recur- sion can be used to prove ELSV-type formulas, such as the original Ekedahl–Lando–Shapiro–Vainshtein formula, the Johnson–Pandharipande–Tseng formula, or Zvonkine’s r-spin formula.

In this talk, it is considered what happens when the y-function of the respective spectral curve acquires logarithmic singularities. Relying on the framework of logarithmic topological recursion (LogTR), I will present a derivation of a new ELSV-type formula for the so-called generalized logarithmic r-spin Hurwitz numbers. I will not discuss the proof of LogTR itself (for these numbers), but rather use it as an input to focus on the geometric and combinatorial techniques required to obtain the respective ELSV-type formula. This is achieved via deforming Zvonkine's qr-ELSV formula in a proper way.

Vasily Gorbounov

Generalized electrical Lie algebras and invariants

My talk is based on joint work with Azat Gainutdinov and Arkady Berenstein, in which we generalize in several ways the electrical Lie algebras originally introduced by Lam and Pylyavskyy. To each semisimple or Kac–Moody Lie algebra g we associate a family of flat deformations of its nilpotent part parametrized by the points of the Cartan subalgebra of g. If $g = sl_n$, then the generic electrical Lie algebra is sp_{n-1}, which is simple if n is odd. Similar situation is with other classical lie algebras, for instance if $g = sp_{2n}$, then its generic electrical Lie algebra is $sp_n \oplus sp_{n-1}$, which is never semisimple.

It turns out (yet conjecturally) that each integrable simple g-module V contains a unique, up to a scalar multiple, electrical invariant, viewed as a deformed highest weight vector. As a consequence, this gives a canonical vector in each weight subspace of V. The conjecture has been proved so far for all classical g.

Alexey Gulyaev

On the prechromatic weight system on primitive elements

In V.A. Vassiliev's theory of finite type knot invariants, the latter are described in terms of weight systems, which are functions on chord diagrams. In the recent work by M. Kazarian, and Z. Yang, the well-known weight systems constructed from the series of Lie algebras gl_N, $N = 1, 2, \ldots$, have been extended from chord diagrams to arbitrary permutations, which al- lowed, in particular, to unify them into a universal gl-weight system. A specialization of this generalization, a polynomial called the prechromatic invariant X, happens to be a one- parameter perturbation of a Hopf algebra homomorphism. The talk is based on our recent paper, where we obtain upper bounds for degrees of terms in the values of X on primitive elements in a Hopf algebra of permutations. The results suggest existence of some algebraic structure on the subspace of primitives. In addition, we describe a class of primitive elements for which the value of X is a homogeneous polynomial. The description is based on a new construction, which associates an oriented knot to each cyclic permutation.

Maxim Kazarian

Relations on kappa-classes and spin Gromov–Witten theory of CP^1

The talk is devoted to the proof of a conjecture by Kazarian–Norbury that states a system of universal polynomial relations among the kappa-classes on the moduli spaces of algebraic curves. The proof involves localization and materialization analysis of the spin Gromov–Witten theory of the projective line. The talk is based on a joint work with A. Alexandrov, B. Bychkov, P. Dunin-Barkowski, and S. Shadrin.

Alexey Kobzev

Ergodic properties of interval exchange transformations

The talk will focus on the ergodic properties of classical interval exchange transformations (IETs) and their generalizations with flips (FIETs) – first return maps for measured foliations on surfaces. The number of intervals indicates the genus of the surface and connects the dynamics with the geometry of moduli spaces.

For IETs, relying on Rauzy cycles, we construct a family with the maximum possible number of ergodic measures $\lfloor n/2 \rfloor$ and obtain two-sided estimates for the Hausdorff dimension of these measures. For FIETs, where a typical system is not minimal, I will present the first irreducible example of a minimal system with exactly three distinct ergodic invariant measures. The construction generalizes M. Keane's method and uses Rauzy induction.

Andrey Mironov

Integrable Birkhoff Billiards inside Cones

We study the integrability of Birkhoff billiards inside cones in R^n. We prove that every trajectory inside a cone over a C^3 strictly convex closed hypersurface embedded in R^{n-1} with non-degenerate second fundamental form has only finitely many reflections. This property allows us to establish that the billiard system is both superintegrable and completely integrable. To the best of our knowledge, this is the first example of an integrable billiard whose table is neither a quadric nor composed of pieces of quadrics. This talk is based on joint works with Siyao Yin.

Oleg Mokhov

Realizations of curvature line nets with a given metric in pseudo-Euclidean spaces and integrable hierarchies

It was previously proven in [1] that any Frobenius manifold (in fact, any solution to the WDVV equations) generates a nonlocal Hamiltonian operator of hydrodynamic type with a flat metric, which is the sum of two compatible Hamiltonian operators – a local one and a purely nonlocal one. These two compatible Hamiltonian operators generate an integrable hierarchy associated with any solution to the WDVV equations (the associativity equations). This talk will present a generalization of this construction to the non-flat case.

[1] Mokhov O.I. Nonlocal Hamiltonian operators of hydrodynamic type with flat metrics, integrable hierarchies, and the associativity equations. Functional Analysis and its Applications. 2006, 40:1, 11-23.

Andrey Okounkov

Quantum cohomology of Hilbert scheme of points in the plane and in the space

Quantum cohomology of Hilbert schemes of points in the n dimensional affine space, when defined, is the basic building block of all-genus curve counting in $n + 1$ folds. For $n = 2$ the story is by now classical, but for $n = 3$ it is new. In my talk I will try explain both, using this example as an introduction to recent joint work with Y. Cao, Y.Zhou, and Z. Zhou.

Vadim Retinskiy

A generalization of the matrix tree theorem to the case of pairs of spanning trees

Matrix tree theorems enumerate spanning trees and spanning forests in various classes of graphs. The first such theorem was obtained by Kirchhoff. It states that the cofactor of any element of the Laplacian matrix of a graph is equal to the number of spanning trees of that graph. Masbaum and Vaintrob associated a skew-symmetric matrix analogous to the Laplacian matrix of an ordinary graph with a 3-graph and proved that the Pfaffian of any principal minor of this matrix enumerates spanning trees in the 3-graph.

In this talk, we present a generalization of the matrix tree theorem to the case of pairs of spanning trees. This result includes as special cases and unifies several previously known theorems, including Kirchhoff's classical matrix tree theorem, the Masbaum–Vaintrob theorem on the enumeration of spanning trees in 3-graphs, and the analogue of the matrix tree theorem recently introduced by Burman and Kulishov.

Michael Shapiro

Poisson–Lie groups and cluster structures

It is well known that cluster structures and Poisson structures in the algebra of regular functions on a quasi-affine variety are closely related. In this talk, I will discuss this connection for Poisson structures defined on a simply connected complex Lie group G by a pair of classical R-matrices. The key element of the construction is a rational Poisson map from the group with a bracket defined by a pair of suitably chosen standard R-matrices to the same group with an arbitrary pair of R-matrices. In the case of $G = \mathrm{SL}_n$, one can build the corresponding cluster structure explicitly and prove its regularity and completeness.

Based on joint work with Misha Gekhtman (Notre Dame) and Alek Vainshtein (University of Haifa).

Ivan Shumilin

Frobenius manifolds constructed from algebro-geometric data and the integrable hierarchies associated with them

The talk is devoted to two integrable hierarchies associated with Frobenius manifolds. The first is constructed from the pencil of flat metrics associated with a Frobenius manifold. The second is governed by a nonlocal Hamiltonian operator determined by the manifold. The relationship between these hierarchies will be discussed in the case of Frobenius manifolds constructed from algebro-geometric data.

Guillaume Tahar

Low dimensional loci in strata of meromorphic differentials

In strata of the moduli spaces of meromorphic differentials, the period coordinate atlas induces, on certain low-dimensional loci, rich geometric structures that often allow for explicit descriptions. In a series of works in collaboration with Dawei Chen, Gianluca Faraco, Quentin Gendron, Myeongjae Lee, Miguel Prado, and Yongquan Zhang, we have given a precise description of several such loci:

- generic isoresidual fibers in strata of differentials with two zeros on the Riemann sphere;
- nilresidual fibers in strata of differentials with one zero on elliptic curves;
- leaves of the isoperiodic foliation in the stratum $H(1, 1, -2)$.

In each case, we will observe a correspondence between the singularities of the geometric structure and the degenerations of the objects parametrized by these moduli spaces.

Vladlen Timorin

Conjugacy invariants of polygon exchange transformations

Polygon exchange transformations (PETs) or, more generally, piecewise isometries (PWIs) are discontinuous dynamical systems that act locally, inside their continuity set, as Euclidean plane isometries. For PETs, these isometries have to be parallel translations. These are 2D generalizations of interval exchange transformations; they appear directly in real-life applications but also as simplified models of more complex dynamical systems. Examples of PWIs are given by outer billiards on polygonal tables. A different viewpoint on PWIs is that these are scissors automorphisms of polygons. Piecewise Euclidean conjugacies between PWIs are then "scissors congruences between scissors automorphisms". This viewpoint allows to apply scissors congruence theory to dynamical systems. One classical example of this approach is given by the Sah–Arnoux–Fathi (SAF) invariant of interval exchange maps. We discuss a generaliza- tion of this approach to PWIs of dimension 2, namely, two different dynamical invariants and their applications to existence of aperiodic points for polygonal outer billiards. One invariant is defined for PETs; it is based on Hadwiger–Glur valuations (which appeared in the solution, by Hadwiger and Glur (1951), of the 2D translation scissors congruence problem). Another invariant is applicable to any orientation preserving PWI; it is based on a certain 1D reduction of a 2D system, for which the SAF invariant is computed. Either of these invariants allows to prove the conjecture posed by R. Schwartz in his ICM-2022 lecture: “Outer billiard on the regular n-gon has an aperiodic orbit unless $n = 3, 4, 6$”. This is a joint project with A. Belyi, A. Kanel-Belov, and Ph. Rukhovich.

Alexandr Veselov

Matrix variations on Moser’s theme

In 1978 Jurgen Moser discovered a remarkable fact: Lagrange multipliers in the classical integrable Neumann system on a sphere turn out to be potentials of finite-gap Schrodinger operators. I will discuss new results linking some integrable systems of classical mechanics with the spectral theory of matrix Schrodinger operators. The talk is based on a joint work with V.E. Adler.

Xin Wang

Universal constraints for Hodge integrals with target varieties and their applications

Motivated by Virasoro conjecture for Gromov–Witten invariants, we propose two conjectures for Hodge integrals with target varieties: Virasoro conjecture and λ_g conjecture. Then we discuss the recent results on these conjectures. As applications, we show how to use these universal constraints to derive new Dubrovin–Zhang type loop equations, which give new algorithms for computing Hodge integrals.

Di Yang

On Legendre-type transformations of a Frobenius manifold

Frobenius manifold was introduced by Dubrovin to give a coordinate-free description of WDVV equations for 2D topological field theories. For a Frobenius manifold M, Dubrovin also introduced Legendre-type transformations $S_\kappa(M)$. In this talk, we show that these Legendre-type transformations $S_\kappa(M)$ share the same monodromy data, and present several applications of it, including the typical example on the relationship between the GUE partition function and the partition function of Gromov–Witten invariants of P^1.

Mikhail Zaitsev

The q-analog of the Argument Shift Method and the Braided Yangian

The classical argument shift method allows one to construct Poisson-commutative subalgebras in the polynomial algebra $S(\mathrm{gl}_N)$, known as Mishchenko–Fomenko algebras. L. Rybnikov proved the commutativity of the lifts of these subalgebras to the universal enveloping alge- bra $U(\mathrm{gl}_n)$. Another family of commutative subalgebras in matrix algebras over $U(\mathrm{gl}_n)$ was in- troduced by Hausel under the name of “big algebras”. In this talk, I will discuss braided analogs of these constructions within the Reflection Equation algebra. To this end, we introduce the notion of quasi-differential operators, which allow us to build braided Hausel and Mishchenko– Fomenko algebras via a procedure that replicates the classical argument shift method. We also define the braided Yangian, which is then used to prove their commutativity.