Mathematics of Multi-Antenna Transmission in 5G networks

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Cellular Networking

2020s: + cloud AR/VR/Gaming, smart cities, robots

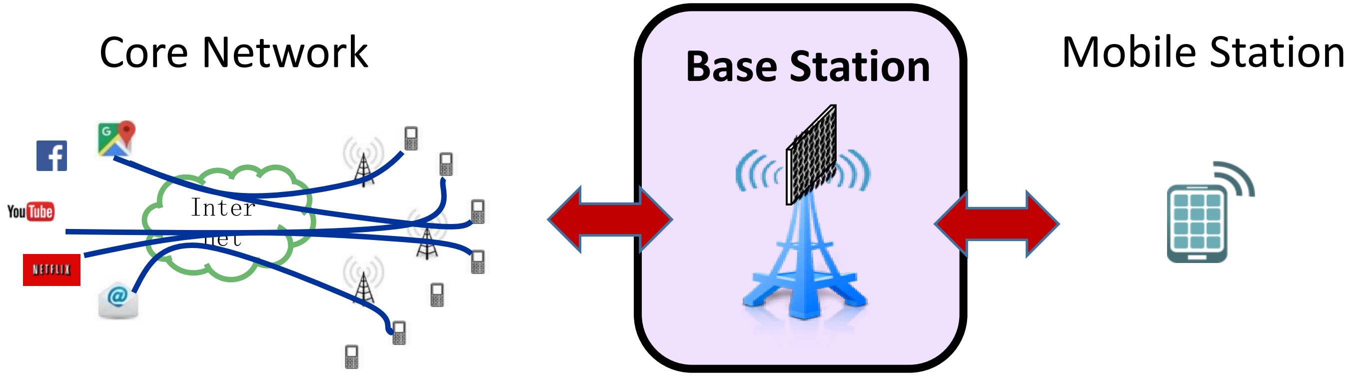
2010s: + video, live streaming, maps

2000s: + web surfing, music

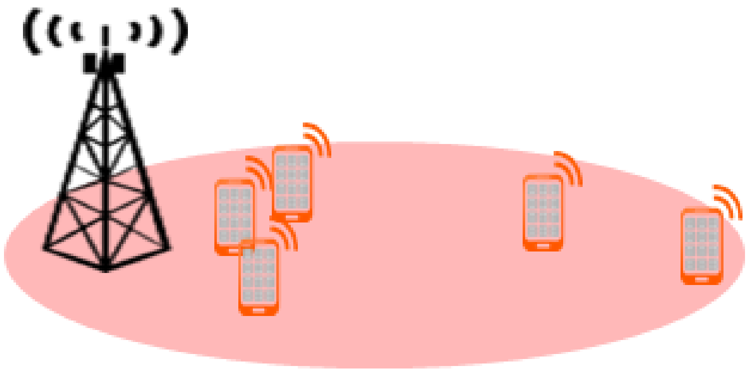
1990s: voice calls, text messaging, basic data service

Evolution mainly due to **mathematics**

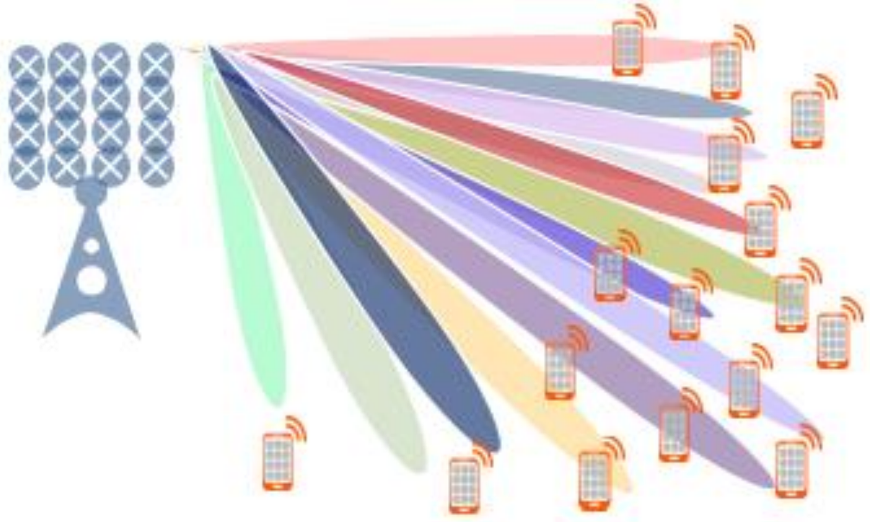
Evolution mainly due to **physics**



Single antenna transmission



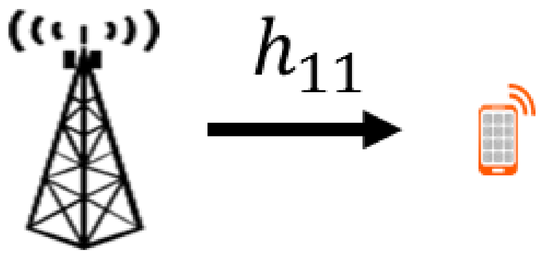
Massive MIMO transmission – key 5G technology



- Evolution due to efficient resource utilization
- **More sophisticated transmission requires cutting-edge mathematical methods**

Source of
- **Non-convex optimization problems**
- **Combinatorial optimization problems**
- **Stochastic optimization problems**
- **Dynamic control problems**

Single antenna transmission



Channel h_{ij}:

$$h_{ij} \in \mathbb{C}, \left|h_{ij}\right| \leq 1$$

$$i \to j$$

Transmitter antenna — Receiver antenna

Describes signal transformation during transmission from antenna i to antenna j

After modulation data is represented by a complex number (symbol):

$$x_1 \in \mathbb{C}$$

$$|x_1|^2 = 1$$

Transmission with power p

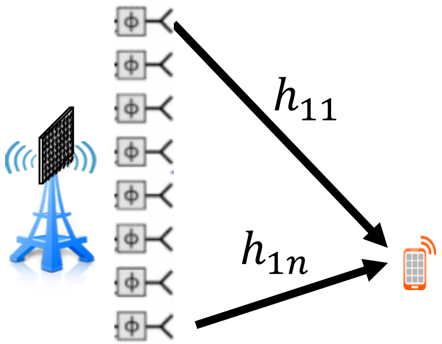
$$y_1 = h_{11} \cdot \sqrt{p} \cdot x_1 + noise$$

Received symbol (y_1) — Transmitted symbol (x_1)

$$SINR_1 = \frac{P(signal)}{P(noise + interference)} = \frac{p \cdot |h_{11}|^2}{\delta^2}$$

Shannon theorem:
upper bound for information transmission capacity is $\log(1 + SINR)$

Multi antenna transmission



n – number of transmitting antennas

w_k^1– "**weight**" of the symbol at antenna

Symbol x_1 is multiplied by w_k^1 and then transmitted from k-th antenna

$$w^1 = \begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \text{ -- precoding vector}$$

$w^1 \in \mathbb{C}^n$,

$||w^1||^2_{L^2} = p$,

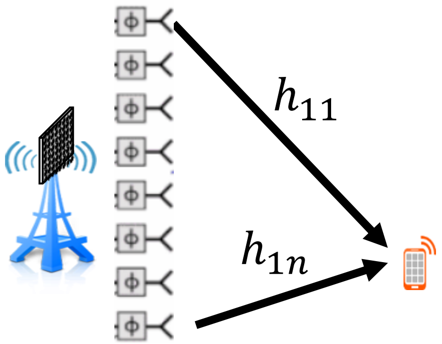
$$y_1 = (h_{11} \dots h_{1n}) \cdot \begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \cdot x_1 + noise$$

$$SINR_1 = \frac{|\langle h_1, {w^1}^* \rangle|^2}{\delta^2}$$

Which **precoding vector** w^1 maximizes $SINR$ for user with channel h_1?

?

Multi antenna transmission



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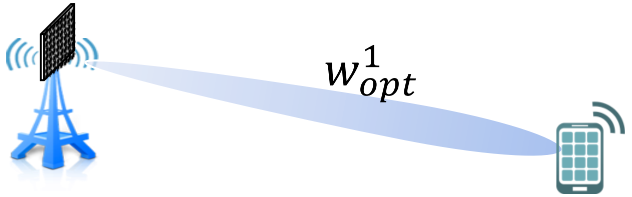
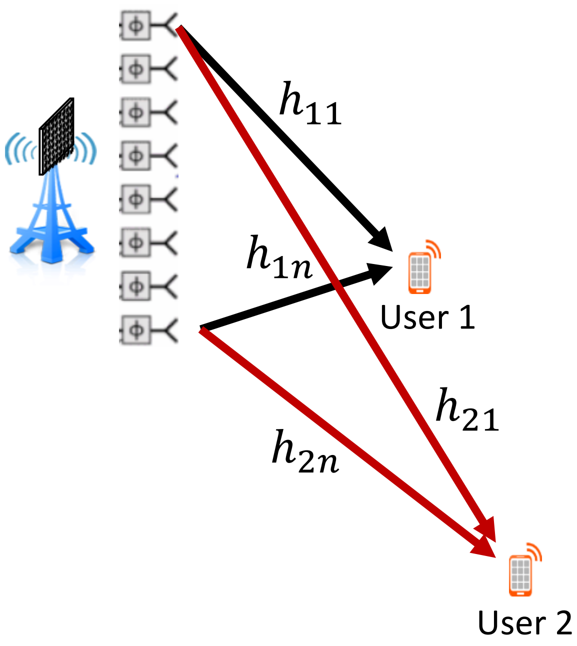
$$SINR_1 = \frac{|\langle h_1, {w^1}^* \rangle|^2}{\delta^2}$$

Which **precoding vector** w^1 maximizes $SINR$ for user with channel h_1?

Answer: $\boldsymbol{w_{opt}^1 = c \cdot h_1^*}$



Multi-user transmission



Transmit two symbols to two different users simultaneously

$$y_1 = (h_{11} \dots h_{1n}) \cdot \left(\begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \cdot x_1 + \begin{pmatrix} w_1^2 \\ \vdots \\ w_n^2 \end{pmatrix} \cdot x_2 \right) + noise_1$$

$$y_2 = (h_{21} \dots h_{2n}) \cdot \left(\begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \cdot x_1 + \begin{pmatrix} w_1^2 \\ \vdots \\ w_n^2 \end{pmatrix} \cdot x_2 \right) + noise_2$$

$$w^k = \begin{pmatrix} w_1^k \\ \vdots \\ w_n^k \end{pmatrix}$$

$w^k \in \mathbb{C}^n$,

$\left\| \sum_k w^k \right\|_{L^2}^2 = p$,

$$y = H \cdot W \cdot x + noise$$

Channel matrix **Precoding matrix**

$$SINR_1(W) = \frac{\left| \langle h_1, w^{1^*} \rangle \right|^2}{\left| \langle h_1, w^{2^*} \rangle \right|^2 + \delta_1^2}$$

$$SINR_2(W) = \frac{\left| \langle h_2, w^{2^*} \rangle \right|^2}{\left| \langle h_2, w^{1^*} \rangle \right|^2 + \delta_2^2}$$

How to choose precoding matrix?

Maximizing weighted sum of spectral efficiency:

$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k) \rightarrow \max_W$$

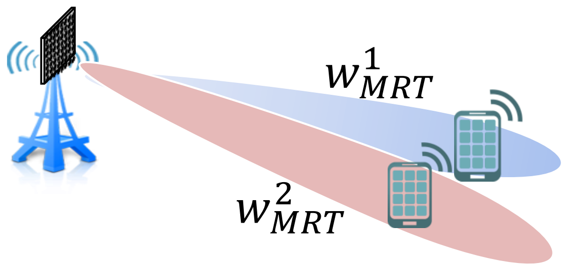
Multi-user beamforming

$$\sum_{u \in U} \alpha_u \cdot \log(1 + SINR_u(W)) \rightarrow \max_W$$

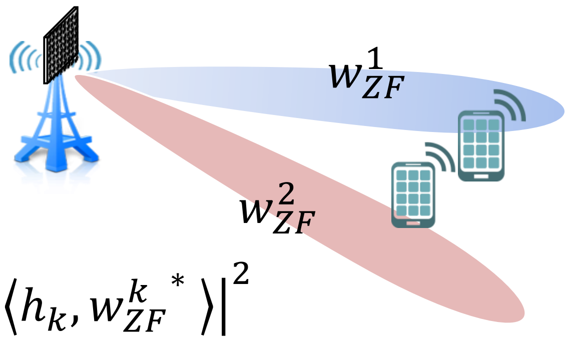
$$SINR_k(W) = \frac{\left|\langle h_k, w^{k^*} \rangle\right|^2}{\left|\langle h_k, w^{l^*} \rangle\right|^2 + \delta_1^2}$$

Classic solution #1: Maximum-rate transmission

$$w^k_{MRT} = c_k \cdot h_k^*$$



Classic solution #2: Zero Forcing



$$SINR_k(W_{ZF}) = \frac{\left|\langle h_k, w^k_{ZF}{}^* \rangle\right|^2}{\delta_1^2}$$

ZF beam orthogonal to all other users channel vectors:

$$w^k_{ZF} \in < h_1, \dots, h_{k-1,} h_{k+1}, \dots, h_n >^{\perp}$$

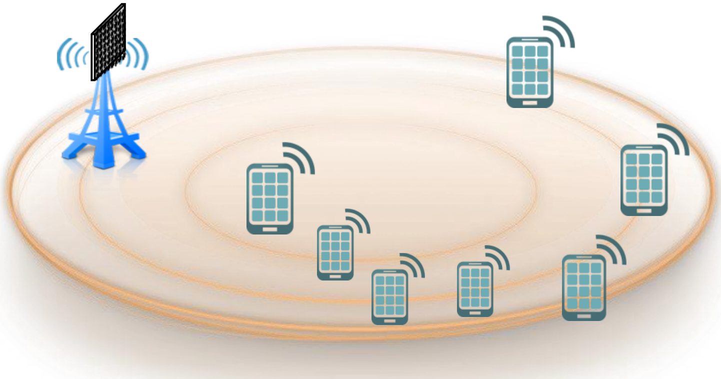
w^k_{ZF} maximizes $\left|\langle h_k, w^k_{ZF}{}^* \rangle\right|^2$ in this subspace

$\mathrm{W_{ZF}}$ is a pseudo-inverse matrix to H:

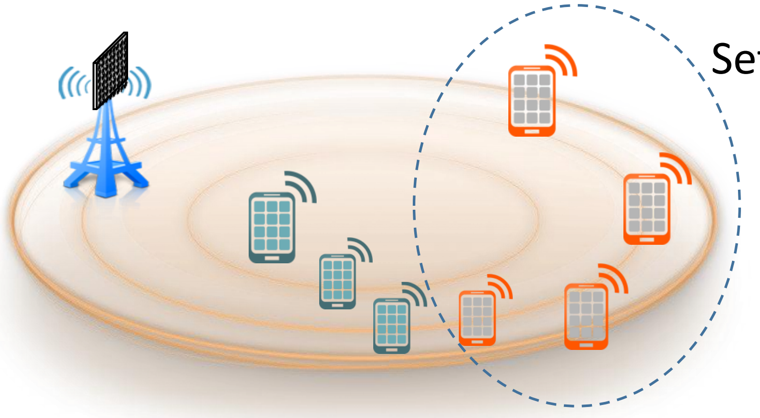
$$\mathrm{W_{ZF}} = H^* \cdot (HH^*)^{-1}$$

Multi-user pairing

Not necessary to transmit to all active users A

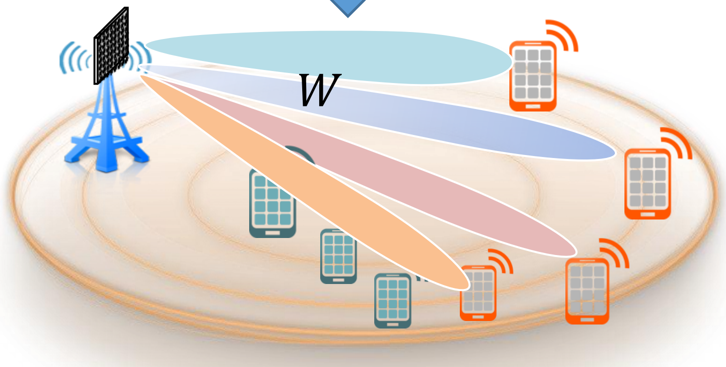


$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k) \rightarrow \max_{U \subseteq A}$$



Set $U \subseteq A$

$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k) \rightarrow \max_{W}$$

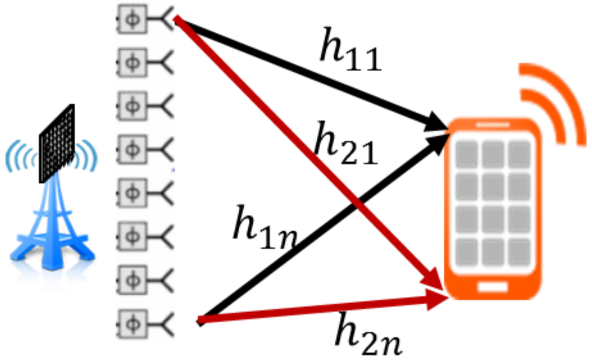


How to select optimal subset $U \subseteq A$ of users for transmission?

$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k(W)) \rightarrow \max_{\substack{U \subseteq A \\ W}}$$

Joint user selection and beamforming?

Multi-stream transmission



$$y_1 = (h_{11} \dots h_{1n}) \cdot \left(\begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \cdot x_1 + \begin{pmatrix} w_1^2 \\ \vdots \\ w_n^2 \end{pmatrix} \cdot x_2 \right) + noise_1$$

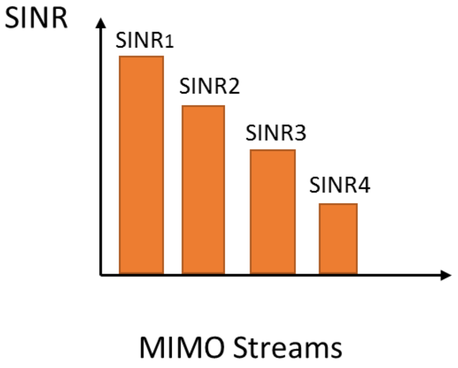
$$y_2 = (h_{21} \dots h_{2n}) \cdot \left(\begin{pmatrix} w_1^1 \\ \vdots \\ w_n^1 \end{pmatrix} \cdot x_1 + \begin{pmatrix} w_1^2 \\ \vdots \\ w_n^2 \end{pmatrix} \cdot x_2 \right) + noise_2$$

$$y = H \cdot W \cdot x + noise$$

Channel matrix **Precoding matrix**

Shannon formula:

$$Rate \leq \log_2 \det \left(I + \frac{P}{N \cdot \delta^2} HW(HW)^* \right)$$



Capacity optimization:
Ideal receiver is assumed

SVD-based Single-User precoding

W is selected as singular vectors of SVD decomposition of H:

$$H = U \cdot \Lambda \cdot V^*$$

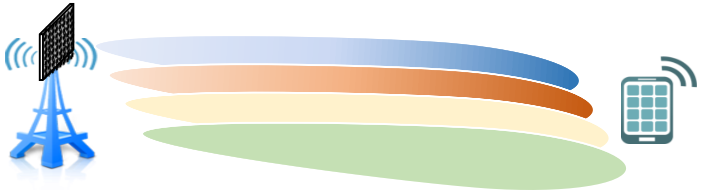
$$W = V$$

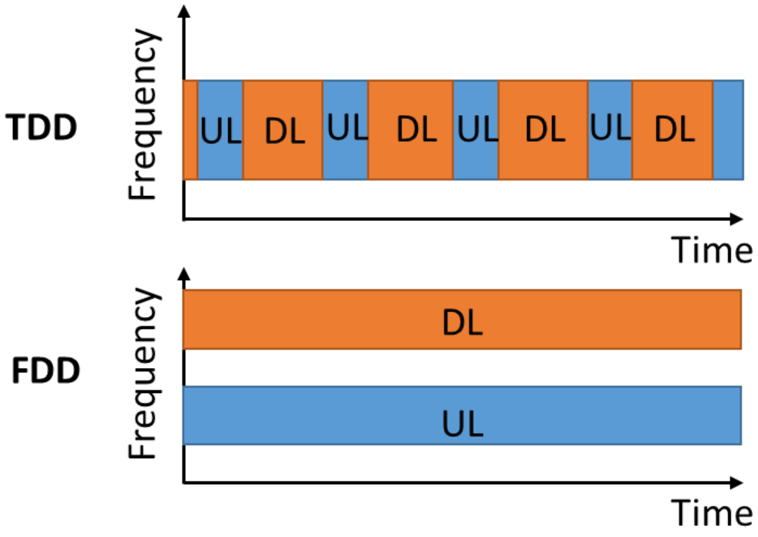
Physical meaning:

1. Choose the best transmitting direction
2. Choose the next best direction in the orthogonal complement to the first one
3. etc…

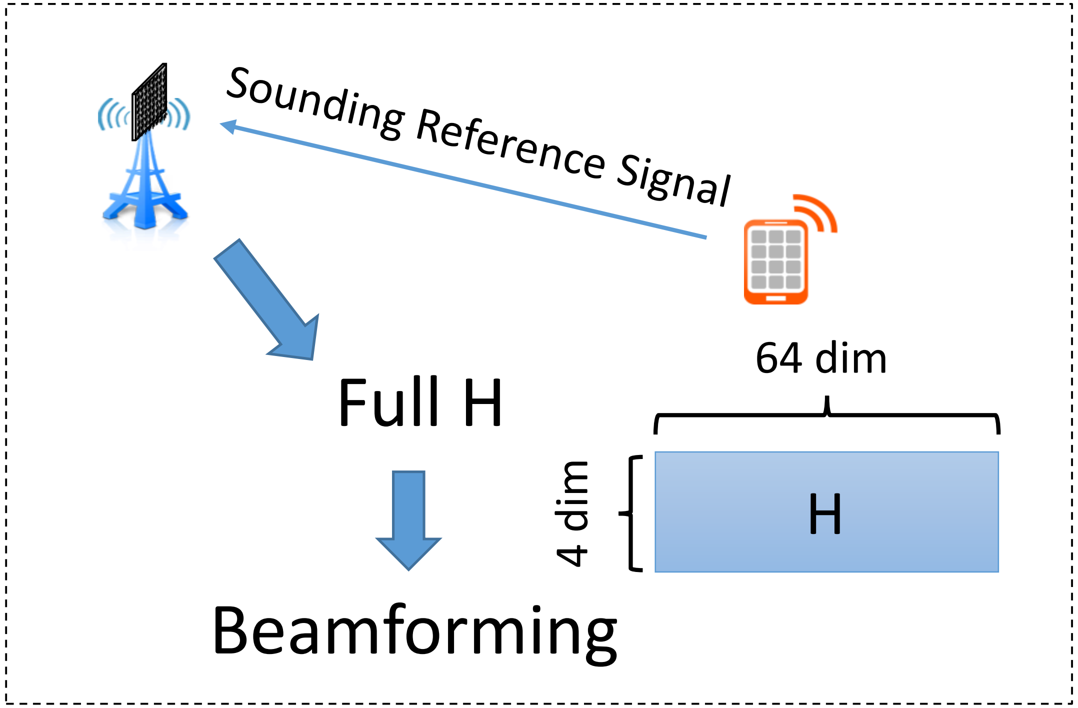
Channel Reconstruction

If we know channel matrix, we may speed up transmission several times:

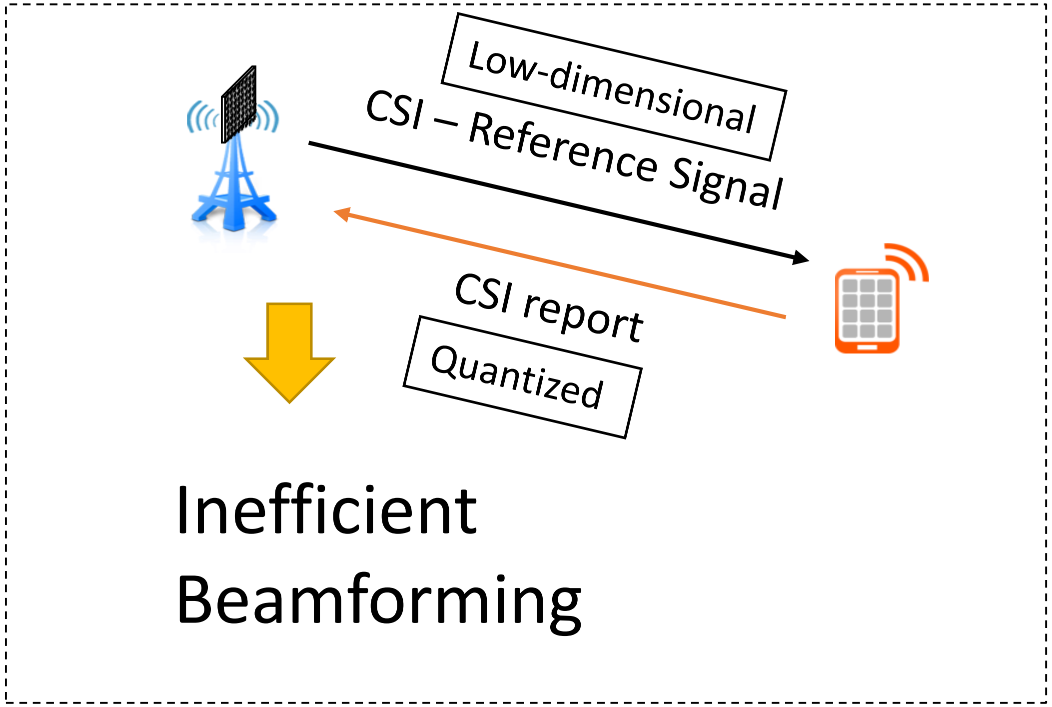




TDD: Uplink Measurement for Downlink Beamforming



FDD: Downlink Measurement and/or Channel Reconstruction



Link adaption problem

MCS – Modulation and Coding Scheme

29 options for MCS

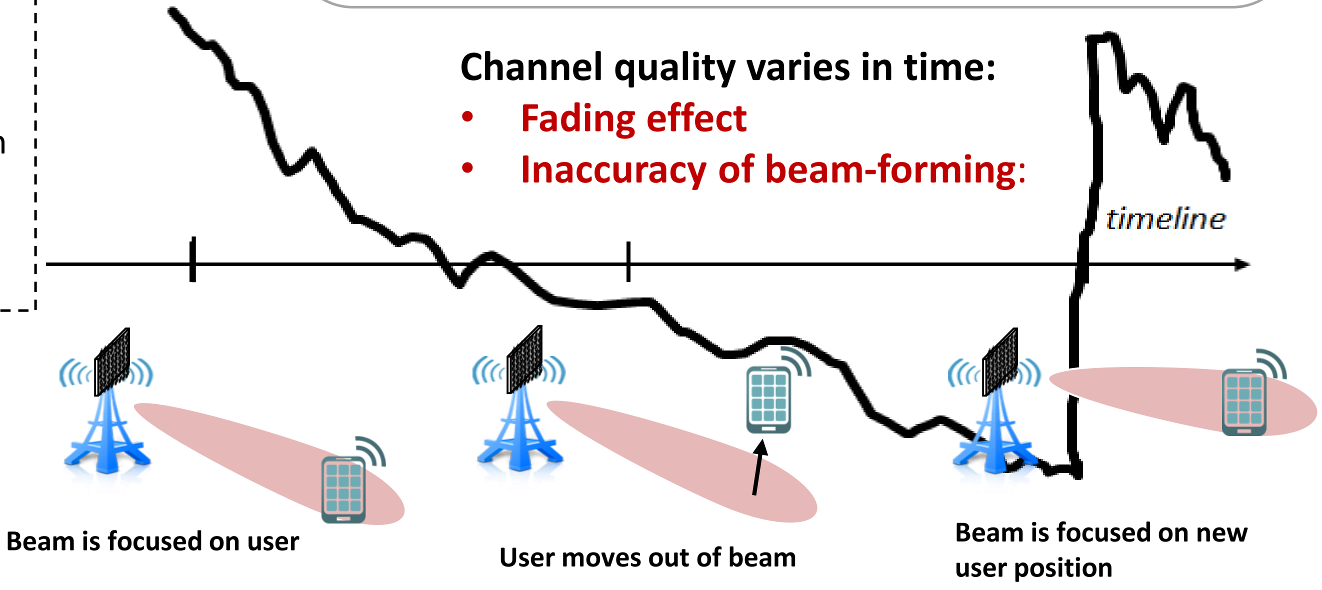
MCS	Modulation	Useful Data 100 PRBs	Total Data 100 PRBs	Code Rate
0	QPSK	2,792	30,000	0.094
1	QPSK	3,624	30,000	0.122
2	QPSK	4,584	30,000	0.154
3	QPSK	5,736	30,000	0.192
4	QPSK	7,224	30,000	0.242
5	QPSK	8,761	30,000	0.293
6	QPSK	10,296	30,000	0.344
7	QPSK	12,216	30,000	0.408
8	QPSK	14,112	30,000	0.471
9	QPSK	15,840	30,000	0.529
10	16 QAM	15,840	60,000	0.264
11	16 QAM	17,658	60,000	0.295
12	16 QAM	19,848	60,000	0.331
13	16 QAM	22,920	60,000	0.382
14	16 QAM	25,456	60,000	0.425
15	16 QAM	28,336	60,000	0.473
16	16 QAM	30,576	60,000	0.510
17	64QAM	30,576	90,000	0.340
18	64QAM	32,856	90,000	0.365
19	64QAM	36,696	90,000	0.408
20	64QAM	39,232	90,000	0.436
21	64QAM	43,816	90,000	0.487
22	64QAM	46,888	90,000	0.521
23	64QAM	51,024	90,000	0.567
24	64QAM	55,056	90,000	0.612
25	64QAM	57,336	90,000	0.637
26	64QAM	61,664	90,000	0.685
27	64QAM	63,776	90,000	0.709
28	64QAM	75,326	90,000	0.837

Modulation: how many bits are encoded into a complex number (4 / 16 / 64)

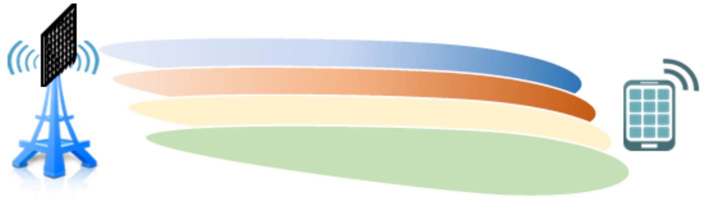
Coding: how many parity bits are used in an error correcting code

MCS should correspond to channel quality.
Higher MCS values

- **allow more information to be transmitted**
- **increase risk of unsuccessful transmission! Retransmissions are painful.**

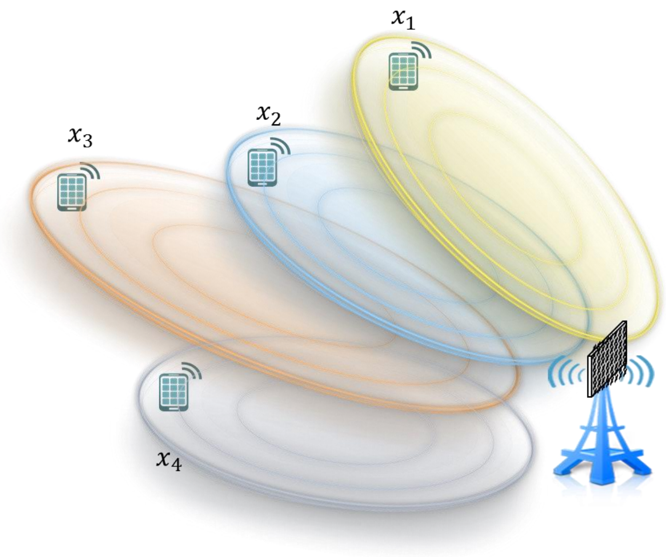


Which number of layers is optimal for transmission?



Which MCS is optimal for transmission?

Uplink Equalization Problem



$$y = H \cdot x + r$$

r **– random noise**
H – channel matrix
x – transmitted signal (dimension = number of users)
y – received signal (dimension = number of base station antennas)

$$\hat{x} = G \cdot H \cdot x + G \cdot r$$

$$y_k = (g_{k1} \dots g_{kn}) \cdot \left(\begin{pmatrix} h_1^1 \\ \vdots \\ h_n^1 \end{pmatrix} \cdot x_1 + \begin{pmatrix} h_1^2 \\ \vdots \\ h_n^2 \end{pmatrix} \cdot x_2 + \cdots + \begin{pmatrix} r_1 \\ \vdots \\ r_n \end{pmatrix} \right)$$

MMSE formulation

$$\boldsymbol{E}_{r,x}\left(\left\|G \cdot y - x\right\|_2^2\right) \to \min_G$$

$$G_{MMSE} = H^* \cdot \left(H \cdot H^* + \frac{\sigma^2}{\rho} \cdot I\right)^{-1}$$

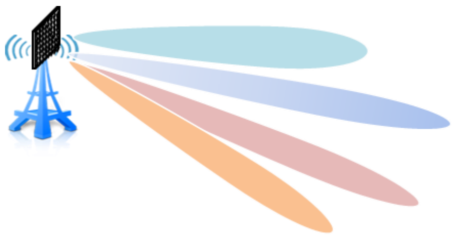
Non-convex formulation

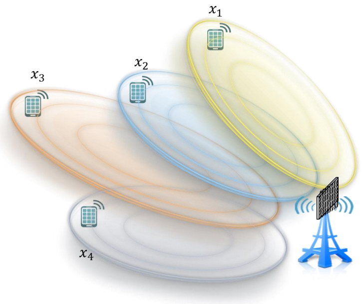
$$\sum_{k \in U} \alpha_k \cdot \log(1 + SINR_k(G)) \to \max_G$$

$$SINR_k(W) = \frac{|\langle g_k, {h_k}^* \rangle|^2}{\left|\langle g_k, h^{l^*} \rangle\right|^2 + |\langle g_k, r^* \rangle|^2}$$

Beamforming problem

- ❖ Non-convex optimization
- ❖ Complex analysis
- ❖ Probability theory



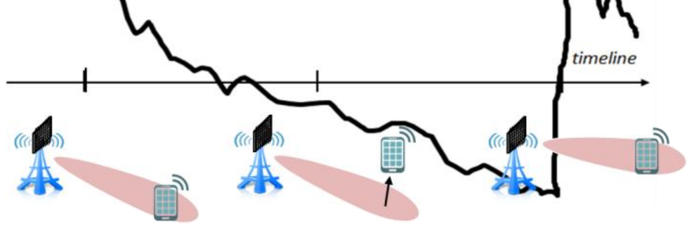


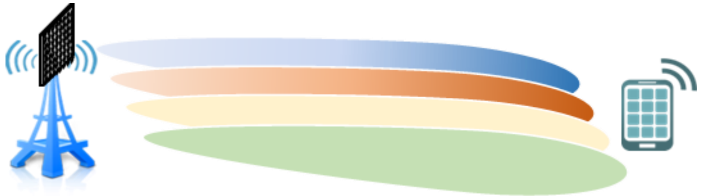
Equalization problem

- ❖ Statistical estimations
- ❖ Probability theory
- ❖ Statistical physics

Link adaption problem

- ❖ Information theory
- ❖ Dynamic control theory
- ❖ Machine learning



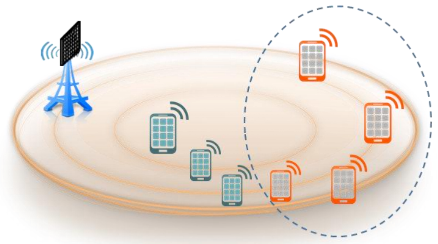


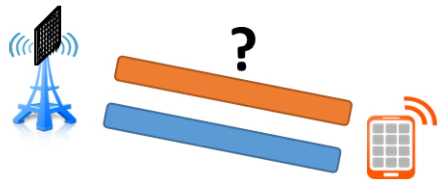
Multi-stream transmission problem

- ❖ Information theory
- ❖ Computational linear algebra
- ❖ Probability theory

User pairing problem

- ❖ Combinatorial optimization
- ❖ Submodular optimization
- ❖ Graph theory





Channel Reconstruction problem

- ❖ Non-convex optimization
- ❖ Complex analysis
- ❖ Stochastic process

Thanks!

My email: zaev.danila1@huawei.com