Optimization Problems of Resource Allocation in 5G Networks

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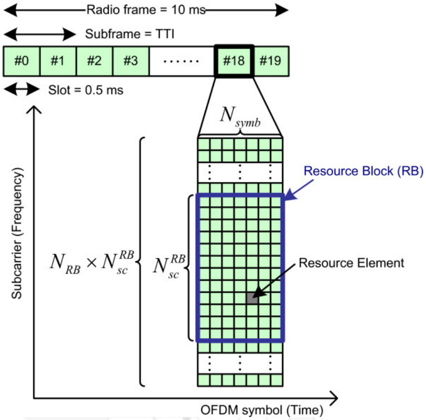
Downlink is a process of data transmission from base station to user equipment (mobile phone) via radio waves

Radio Resources



- **TTI** means transmission time interval;
- **Radio resource element** is an interval in the time-frequency domain used for transmission of a single symbol;
- resource elements are grouped into blocks (**RB**) which are also combined into groups (**RBG**);
- radio resources are being allocated by groups;
- the more radio resources are allocated for one user, the more data may be transmitted to him;
- in 5G one radio resource may be assigned for different users (**multi-user pairing**);

Image source: https://www.netmanias.com/en/post/blog/11339/lte/lte-and-beyond-dl-throughput-comprehensive-calculations

MU-MIMO User Scheduler in Downlink

Multi-User Multiple-Input and Multiple-Output

The task of the scheduler is to distribute RBGs over users every TTI.

user index, j

RBG index, k	0	1	2	3	4	...	29
0	✓			✓		...	✓
1		✓	✓		✓	...	
2	✓		✓		✓	...	✓
3		✓				...	
...	...	...	...	...	...	...	...
16		✓		✓	✓	...	✓

What is the optimality criterion for this procedure?

Let $x_j^k(t)$ be a boolean pairing decision variable:

$x_j^k(t) = 1 \Leftrightarrow$ the RBG k is assigned for the user j at the TTI t. $\mathbf{x}(t) = \left(x_j^k(t)\right)_{j,k}$

$A^k(\mathbf{x}(t)) := \{j | x_j^k(t) = 1\}$ is the set of users on the RBG k at the TTI t

$B_j(\mathbf{x}(t)) := \{k | x_j^k(t) = 1\}$ is the set of RBGs assigned for the user j at the TTI t

user index, j

RBG index, k	0	1	2	3	4	...	29
0	1	0	0	1	0	...	1
1	0	1	1	0	1	...	0
2	1	0	1	0	1	...	1
3	0	1	0	0	0	...	0
...	...	...	...	...	...	...	...
16	0	1	0	1	1	...	1

For this example $A^2 = \{0, 2, 4, \ldots, 29\}$, $B_3 = \{0, \ldots, 16\}$

Transport Block Size Estimation

$\texttt{TBS}_j(t)$ is an amount of useful data sent to the user j at the TTI t.

$$A^k(\mathbf{x}(t)) := \{j | x_j^k(t) = 1\};\ B_j(\mathbf{x}(t)) := \{k | x_j^k(t) = 1\};$$

$$\boxed{\texttt{TBS}_j(t, \mathbf{x}(t)) \propto |B_j(\mathbf{x}(t))| \cdot \log_2\left(1 + \texttt{SINR}_j(t, \mathbf{x}(t))\right);}$$

$\texttt{SINR}$ means signal-to-interference-plus-noise ratio

$$\texttt{SINR}_j(t, \mathbf{x}(t)) := \mathrm{mean}\{\texttt{SINR}_j^k(t, \mathbf{x}(t)) | k \in B_j(\mathbf{x}(t))\};$$

$$\texttt{SINR}_j^k(t, \mathbf{x}(t)) := \frac{\left|\left[\hat{H}^k(t, \mathbf{x}(t))\right]_j^j\right|^2}{\sum_{j' \in A^k(\mathbf{x}(t)) \setminus \{j\}} \left|\left[\hat{H}^k(t, \mathbf{x}(t))\right]_{j'}^j\right|^2 + \sigma_j^2(t) \cdot |A^k(\mathbf{x}(t))|},$$

where σ_j^2 is the noise power on antennas of the user j,
$\hat{H}^k(t, \mathbf{x}(t))$ is a Hermitian matrix of size $|B^k(t)|$

MU Pairing Problem

One of possible optimality criteria is the **perceived throughput** functional:

$$F[\mathbf{x}] := \frac{\sum_{t,j} \min\{\mathtt{TBS}_j(t, \mathbf{x}(t)), \mathtt{Buffer}_j(t)\}}{\sum_j T_j(\mathbf{x})} \to \max_{\mathbf{x}} \; s.t. \; \sum_j x_j^k(t) \le L \;\; \forall k, t$$

$\mathtt{TBS}_j(t)$ is the transport block size of the user j (defined previously)
$\mathtt{Buffer}_j(t)$ is an amount of data requested by the user j at the TTI t
$T_j(\mathbf{x})$ is the number of TTIs when user j has non-empty buffer
L is the maximal possible number of users on RBG

The difficulty is that we **do not know in advance** how TBS and buffer depend on time.

In what sense solution would be optimal?

Single-TTI Approach

In order to simplify the problem one can consider it on a single TTI (t is frozen), i.e. solve the following optimization problem:

$$F(\mathbf{x}) := \sum_j \alpha_j \min\{\mathtt{TBS}_j(\mathbf{x}), \mathtt{Buffer}_j\} \to \max_{\mathbf{x}} \;\; s.t. \; \sum_j x_j^k \le L \quad \forall k,$$

where α_j is a priority value of the user j (some predefined parameter)

Can we choose α_j such that single-TTI optimization will provide a suboptimal solution for the initial cross-TTI problem?

What properties does this function have? How to optimize it?

Further Simplification

Let us suppose that we have only one RBG ($k \in \{0\}$) and all users have extremely large buffer. In this case our purpose is to maximize the following function:

$$F^o(\mathbf{x}) := \sum_{j \in A(\mathbf{x})} \alpha_j \log_2 \left(1 + \frac{\left| \left[\hat{H}(\mathbf{x}) \right]_j^j \right|^2}{\sigma_j^2 \cdot |A(\mathbf{x})| + \sum_{j' \in A(\mathbf{x}) \setminus \{j\}} \left| \left[\hat{H}(\mathbf{x}) \right]_{j'}^j \right|^2} \right)$$

under the same constraint $|A(\mathbf{x})| \leq L$

As $A(\mathbf{x}) = \{j | x_j^0 = 1\}$, $\mathbf{x}$ is the indicator vector of the set $A(\mathbf{x})$ and so the function $F^o(\mathbf{x})$ is a set function of $A(\mathbf{x})$

What properties does this function have? How to optimize it?

Let U be the set of all users.

- $F: 2^U \to \mathbb{R}$ is called **nonnegative** iff for any $A \subset U$ $F(A) \geq 0$
- $F: 2^U \to \mathbb{R}$ is called **monotone** iff for any $A_1 \subset A_2 \subset U$

$$F(A_1) \leq F(A_2)$$

«The larger the set the larger the function value»

- $F: 2^U \to \mathbb{R}$ is called **submodular** iff for any $A_1 \subset A_2 \subset U$, $j \in U \backslash A_2$

$$F(A_1 \cup \{j\}) - F(A_1) \geq F(A_2 \cup \{j\}) - F(A_2)$$

«The larger the set the smaller the gain»

- $F: 2^U \to \mathbb{R}$ is called **DS** iff it can be represented as a difference of two submodular functions

Theorem. Any set function is DS. [Narasimhan, Bilmes, 2005]

Greedy Algorithm for Submodular Optimization

$$\Delta F(j|A) := \begin{cases} F(A \cup \{j\}) - F(A), \ j \notin A; \\ F(A \backslash \{j\}) - F(A), \ j \in A; \end{cases}$$

```
A_gr := ∅;
while True do
    j' := argmax_j ΔF(j|A_gr);
    if ΔF(j|A_gr) > 0 then
        A_gr := { A_gr ∪ {j'}, j' ∉ A_gr;
                { A_gr \ {j'}, j' ∈ A_gr;
    else
        return A_G;
    end
end
```

- $F(A_{gr}) \geq (1 - \frac{1}{e})F(A_{opt})$ for <u>monotone</u> submodular set functions under constraint $|A| \leq n < |U|$ (home exercise)
- $F(A_{gr}) \geq \frac{1}{3}F(A_{opt})$ for nonnegative submodular set functions [Feige et. al., 2011]

DS Optimization

Let G', G'' be submodular set functions such that $F^o(A) = G'(A) - G''(A)\ \forall A \subset U$.

$$G''_C(A) := G''(C) - \sum_{j \in C \setminus A} \left(G''(C) - G''(C \setminus \{j\})\right) + \sum_{j \in A \setminus C} G''(\{j\});$$

G''_C is called modular upper bound of G'' at C with the following properties:

- $G''_C(A) \geq G''(A)\ \forall A \subset U$;
- $G''_C(C) = G''(C)$;
- both G''_C and $-G''_C$ are submodular.

$F(A) = G'(A) - G''(A) \geq G'(A) - G''_C(A)$ – the r.h.s. is a submodular function

while $A_t \neq A_{t-1}$ **do**
  $A_{t+1} := \mathrm{argmax}_A \left(G'(A) - G''_{A_t}(A)\right)$;
end

[Iyer, Bilmes, 2013]
Can we design a simple heuristics based on this approach?

Thank you!

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