EXERCISES

1. List all clusters in the cluster structure associated with the homogeneous coordinate ring of $Gr_2(5)$.

2. List all elements of an additive basis of homogeneous polynomials in the Plücker coordinates x_{ij} of degree 2 in the homogeneous coordinate ring $\mathbb{C}[Gr_2(5)]$.

3. Find the number of connected components of the set of complete real flags in $\mathbb{R}^4$ transversal to a given pair of mutually transversal complete flags.

4. Check that the oriented medial graph of triangulation of pentagon changes according to the mutation rule under flip.

5. **Definition**. An integer $n \times n$ matrix B is left (right) skew-symmetrizable if there is an integer diagonal $n \times n$ matrix D such that DB (BD, correspondingly) is skew-symmetric.

Show that any left skew-symmetrizable matrix is also right skew-symmetrizable.

6. Check that the form $\omega = \sum_{ij} b_{ij} \frac{dx_i}{x_i} \wedge \frac{dx_j}{x_j}$ is closed, i.e. $d\omega = 0$. Show that distribution $Ker(\omega)$ is integrable, i.e. there is a submanifold S in $\mathbb{A}^n$ of dimension $\dim Ker(\omega)$ such that the tangent space $T_x(S)$ at each point $x \in S$ coincides with $Ker(\omega)|_x$.

7. Let rational functions $\{f_i(x_1, \ldots, x_n)\}_{1 \le i \le n}$ be generators of the field of rational functions $\mathbb{C}(x_1, \ldots, x_n)$, $(\omega_{ij})_{1 \le i,j \le n}$ be an skew-symmetric matrix. Then, $\{f_i, f_j\} = \omega_{ij} f_i f_j$ determines a Poisson bracket on $\mathbb{A}^n$.

8. Check that diagram mutation is well defined.

9. Check that the mutation class of the following matrix

$$\begin{pmatrix} 0 & 2 & -4 \\ -1 & 0 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$

is finite.