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Laboratory on Algebraic Transformation Groups

Moscow, 2021

Research areas

Laboratory on Algebraic Transformation Groups was created at the Faculty of Mathematics of HSE in June 2020.

- Algebraic Geometry
- Commutative Algebra
- Invariant Theory
- Representation Theory
- Lie Groups and Lie Algebras

Main research topics

- Automorphism groups of algebraic varieties
- Varieties with torus actions and graded algebras
- Additive actions on complete varieties
- Locally nilpotent derivations
- Cox rings and their applications

Goals of the laboratory

- Study of algebraic transformation groups

- Involvement of students and postgraduates in research activities and current projects of the laboratory

- Organization of conferences, schools, and seminars in affine algebraic geometry and transformation groups

Researchers

We have 1 Professor, 3 Research Fellows and 11 Research Assistants working in our Laboratory.

Research Assistants

Yulia Zaitseva
HSE
Doctoral School in Mathematics, postgraduate
Lecturer at HSE

Boris Bilich
HSE
Faculty of Mathematics, 4th year student

Alisa Chistopolskaya
HSE
Faculty of Mathematics, 4th year student

Viktoriia Borovik
MSU
Faculty of Mechanics and Mathematics, 5th year student

Alexander Popkovich
HSE
Faculty of Mathematics, master student

Grigory Taroyan
HSE
Faculty of Mathematics, 4th year student

Sergey Dzhunusov
MSU
Faculty of Mechanics and Mathematics, graduate

Evgeny Zhukov
HSE
Faculty of Mathematics, 4th year student

Denis Kremko
HSE
Faculty of Mathematics, 3rd year student

Nikolai Slinkin
HSE
Faculty of Mathematics, master student

Kirill Shakhmatov
MSU
Faculty of Mechanics and Mathematics, 5th year student

Research Fellows

Sergey Gaifullin

2012 Candidate of Sciences (PhD) in Mathematics
Lomonosov Moscow State University

- Associate Professor at MSU
- Associate Professor at HSE

Alexander Perepechko

2014 Candidate of Sciences (PhD) in Mathematics
Lomonosov Moscow State University and University of Grenoble

- Associate Professor at MIPT
- Researcher at Kharkevich Institute

Anton Shafarevich

2019 Candidate of Sciences (PhD) in Mathematics
Lomonosov Moscow State University

- Teaching assistant at MSU
- Teaching assistant at HSE

Ivan Arzhantsev

Laboratory Head, Senior Research Fellow

Author of over 50 scientific papers and 4 manuals. Co-author of a 2015 monograph in the Cambridge Studies in Advanced Mathematics series. He has trained 10 candidates of sciences.

- Dean of the Faculty of Computer Science
- Professor of the Faculty of Computer Science/Big Data and Information Retrieval School
- Member of the HSE Academic Council
- Senior Research Fellow of the Faculty of Mathematics

Degrees and Academic Titles

2020 Professor

2011 Doctor of Sciences in Mathematical Logic, Algebra and Number Theory
Lomonosov Moscow State University
Thesis Title: *Embeddings of Homogeneous Space and Geometric Invariant Theory*

2009 Associate Professor

1998 Candidate of Sciences (PhD) in Mathematical Logic, Algebra and Number Theory
Lomonosov Moscow State University
Thesis Title: *Reductive group actions with spherical orbits*

1995 Degree in Mathematics. Applied Mathematics
Lomonosov Moscow State University

Seminars organized by the laboratory

- Seminar on Additive Actions (2019-2020)

over 30 reports on theory of additive actions, local algebras, toric geometry, algebraic number theory.

- Seminar on Algebraic Transformation Groups (2020-present)

main topics discussed in the seminar: toric geometry, theory of locally nilpotent derivations, infinite transitivity, infinite-dimensional algebraic geometry, theory of spherical varieties, infinite dimensional Lie algebras.

Schools and conferences supported by the laboratory

Past Events

- Topology and Geometry of Group Actions
  - 18–22 November 2020, HSE University, Moscow, Russia

- Lie algebras, Algebraic Groups and Invariant Theory
  - 27 January – 1 February 2020, Moscow State University, Steklov Mathematical Institute, Moscow, Russia

Schools and conferences organized by the laboratory

Forthcoming Events

- Affine Varieties: Embeddings, Automorphisms, Structure and Topology (AV-EAST-2021)

  Dedicated to 70th anniversary of Shulim Kaliman

  - 26–30 July 2021, Euler Institute, Saint Petersburg, Russia

At Summer School "Visions of Algebraic Groups", Euler Institute, Saint Petersburg, August 2019

Our partners

- Moscow State University, Moscow, Russia
- Steklov Mathematical Institute of Russian Academy of Science, Moscow, Russia
- The Euler International Mathematical Institute, St. Petersburg, Russia
- Samara University, Samara, Russia
- Fourier Institute, Grenoble, France
- Eberhard Karls University of Tübingen, Germany

Grants

- **Russian Foundation for Basic Research**, **grant "Expansion"**, project 20-11-50106 "Automorphisms of algebraic varieties and infinite transitivity", 2020-2021, Head — Ivan Arzhantsev.
- **Basis Foundation**, **grant “Leader”**, project “Automorphisms of affine varieties, graded algebras and locally nilpotent derivations”, 2018-2021, Head — Ivan Arzhantsev.
- **Russian Science Foundation**, **grant 20-71-00109**, project “Orbits of regular automorphisms groups of G-varieties”, 2020-2022, Head — Sergey Gaifullin.
- **Russian Science Foundation**, **grant 19-11-00172**, project “Additive actions on complete algebraic varieties and their generalizations”, 2019-2021, Head — Ivan Arzhantsev.

Awards

- **Sergey Dzhunusov** was awarded a finalist diploma in the "Undergraduates" category of the 24nd August Möbius Competition 2020 for his work *«Additive actions on complete toric surfaces and On uniqueness of additive actions on complete toric varieties».*

- **Yulia Zaitseva** was awarded a second degree diploma in the "Undergraduates" category of the 22nd August Möbius Competition 2018 for her work *"Homogeneous locally nilpotent derivations of non-factorial trinomial algebras".*

- **Dmitry Matveev** was awarded a finalist diploma in the "Undergraduates and graduates" category of the 22nd August Möbius Competition 2018 for his work *"Commuting homogeneous locally nilpotent derivations".*

- **Yulia Zaitseva** was awarded a finalist diploma in the "Undergraduates" category of the 23nd August Möbius Competition 2019 for her work *"Commutative algebraic monoid structures on affine spaces".*

Talks

- Topology and Geometry of Group Actions, November 2020, Moscow.
    - Talk *Automorphisms of affine surfaces, the Markov numbers, and the Thompson group T.* **Alexander Perepechko.**

- Zoom Algebraic Geometry Marathon, September 2020.
    - Talk *Generic flexibility of cubic cones*. **Alexander Perepechko.**

- Workshop on Torus Actions in Topology, May 2020, Toronto.
    - Talk *Infinite transitivity, finite generation, and Demazure roots.* **Ivan Arzhantsev.**
- Iskovskikh Seminar, March 2020, Moscow.
    - Talk *Commutative algebraic monoid structures on affine spaces.* **Yulia Zaitseva.**

- The eighth school-conference on Lie Algebras, Algebraic groups and Invariant Theory, January 2020, Moscow.
    - Talk *Additive actions on complete algebraic varieties.* **Ivan Arzhantsev.**
    - Talk *Additive actions on toric projective hypersurfaces*. **Anton Shafarevich.**
    - Poster *Commutative actions on smooth quadrics.* **Viktoriia Borovik, Sergey Gaifullin, Anton Trushin.**
    - Poster *Additive actions on complete toric surfaces*. **Sergey Dzhunusov.**
    - Poster *Nilpotent generators of the symplectic Lie algebra.* **Alisa Chistopolskaya.**
    - Poster *Commutative algebraic monoid structures on affine spaces.* **Yulia Zaitseva.**

Publications

More than 20 articles have been prepared in 2020-2021. Eight of them have already been published in Q1 and Q2 journals.

Recent publications

- Ivan Arzhantsev and Mikhail Zaidenberg. Tits alternative and highly transitive actions on toric varieties. International Mathematics Research Notices Published online (2021). Q1 SJR.
- Ivan Arzhantsev, Alvaro Liendo, and Taras Stasyuk. Lie algebras of vertical derivations on semiaffine varieties with torus actions. Journal of Pure and Applied Algebra 225 (2021), no. 2, 106499: 1-18. Q1 SJR.
- Sergey Gaifullin. Automorphisms of Danielewski varieties. Journal of Algebra 573 (2021), 364-392. Q1 SJR.
- Sergey Dzhunusov and Yulia Zaitseva. Commutative algebraic monoid structures on affine surfaces. Forum Mathematicum 33 (2021), no. 1, 177-191. Q1 SJR.
- Alexander Perepechko. Affine cones over cubic surfaces are flexible in codimension one. Forum Mathematicum, Published online (2020). Q1 SJR.
- Ivan Arzhantsev, Sergey Bragin, and Yulia Zaitseva. Commutative algebraic monoid structures on affine spaces. Communications in Contemporary Mathematics 22 (2020), no. 8, 1950064: 1-23. Q1 SJR.
- Alisa Chistopolskaya. On nilpotent generators of the symplectic Lie algebra. Published online in Linear and Multilinear Algebra (2020). Q2 SJR.
- Sergey Dzhunusov Additive actions on complete toric surfaces. International Journal of Algebra and Computation. Published online (2020). Q2 SJR.

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Laboratory on Algebraic Transformation Groups

Some results

Commutative algebraic monoids

an algebraic variety X with a commutative associative multiplication $X \times X \to X$, which is a morphism and has a unit.

The case of affine space $X = \mathbb{A}^n$

Ivan Arzhantsev, Sergey Bragin, and Yulia Zaitseva. Commutative algebraic monoid structures on affine spaces. Communications in Contemporary Mathematics (2020), Q1 SJR

$\mathbb{A}^1$: $x \cdot y = xy$ $\qquad$ $x \cdot y = x + y$

$\mathbb{A}^2$:
$$(x_1, x_2) \cdot (y_1, y_2) = (x_1 y_1, x_2 y_2) \qquad (x_1, x_2) \cdot (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$
$$(x_1, x_2) \cdot (y_1, y_2) = (x_1 y_1, x_1^b y_2 + y_1^b x_2)$$

$\mathbb{A}^3$:
$$(x_1, x_2, x_3) \cdot (y_1, y_2, y_3) = (x_1 y_1, x_2 y_2, x_3 y_3)$$
$$(x_1, x_2, x_3) \cdot (y_1, y_2, y_3) = (x_1 y_1, x_2 y_2, x_1^b x_2^c y_3 + y_1^b y_2^c x_3)$$
$$(x_1, x_2, x_3) \cdot (y_1, y_2, y_3) = (x_1 y_1, x_1^b y_2 + y_1^b x_2, x_1^c y_3 + y_1^c x_3 + Q(x_1, y_1, x_2, y_2))$$

The case of affine surface X

Sergey Dzhunusov and Yulia Zaitseva. Commutative algebraic monoid structures on affine surfaces. Forum Mathematicum (2021), Q1 SJR

$$\chi^u \to \chi^u \otimes \chi^u (1 \otimes \chi^u + \chi^u \otimes 1)^{<p,u>}$$

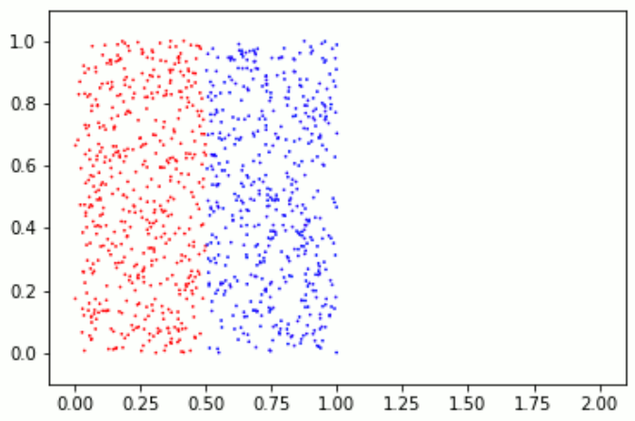
Infinite transitivity

- Physical picture: a configuration of particles in a state-space. Suppose that some forces are acting on these particles. When all configurations with a given number of particles **are equivalent** under the action of these forces?

- Assume that the state-space is an *algebraic variety* and the forces are acting *algebraically.* Then the problem can be approached using the theory of algebraic transformation groups.

Ivan Arzhantsev. Infinite transitivity and special automorphisms.
Arkiv for Matematik (2018), Q2 SJR

- **Infinitely transitive action** of a group is a purely algebraic substitute for the concept of a **mixing dynamical system.** The action is infinitely transitive precisely when a configuration of non-coinciding points is equivalent to any other such configuration.



Ivan Arzhantsev, Karine Kuyumzhiyan, and Mikhail Zaidenberg.
Infinite transitivity, finite generation, and Demazure roots
Advances in Mathematics (2019), Q1 SJR

- Currently the researchers Alisa Chistopolskaya and Grigory Taroyan are preparing a paper on the partial solution of this problem in 2-dimensional case.

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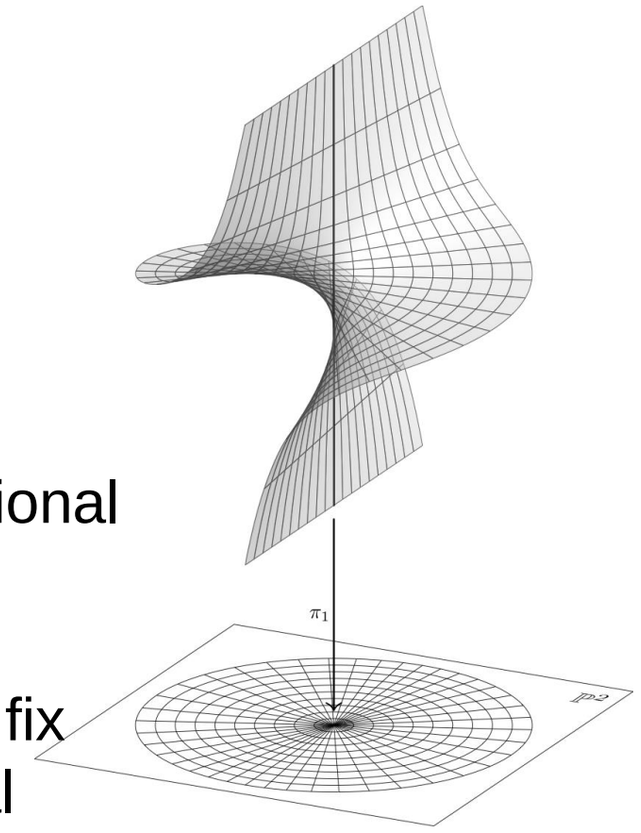
Affine cones over cubic surfaces

Cubic surfaces are blowups of the projective plane in 6 points. They contain 27 lines.

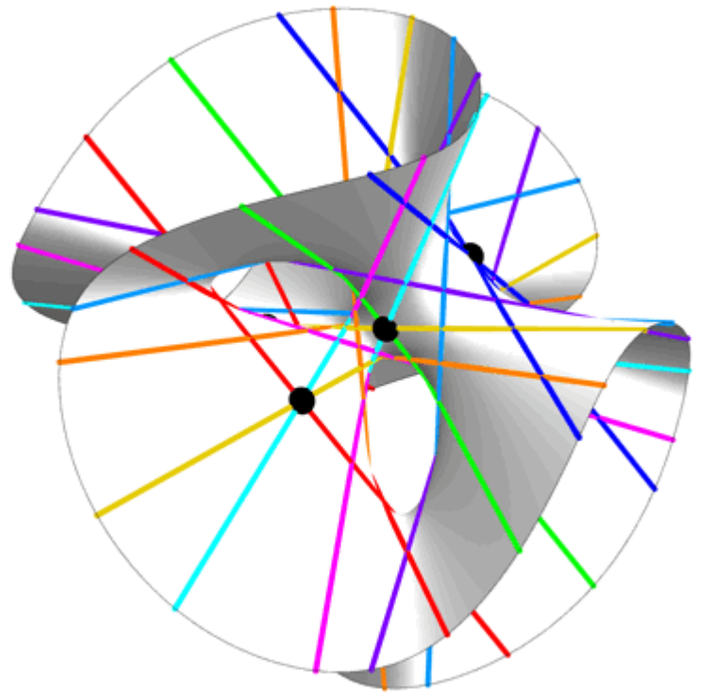
The **affine cones** over them are described by points in the 7-dimensional polyhedral cone.

The transformations of affine cones fix some of lines and exist in polyhedral subcones, which we studied using **SageMath**.

We proved that all the affine cones admit infinite transitivity almost everywhere.



Blowup at a point
Image by Charles Staats



Clebsch cubic surface
$x_1^3+x_2^3+x_3^3+x_4^3=(x_1+x_2+x_3+x_4)^3$
Image by Greg Egan

Alexander Perepechko. Affine cones over cubic surfaces are flexible in codimension one. Forum Mathematicum (2021), Q1 SJR

Automorphisms of Danielewski varieties

Danielewski variety $xy_1^{k_1}y_2^{k_2}\dots y_m^{k_m} = P(z, y_1, \dots, y_m)$, where $k_i \geq 2$, $P(z, y_1, \dots, y_m) = z^d + s_{d-1}(y_1, \dots, y_m) + \dots + s_0(y_1, \dots, y_m), d \geq 2.$

These varieties provide a counter-example to generalized Zariski problem.

THEOREM.

Let G be the subgroup of automorphisms that permute y_i with coinciding k_i and multiply variables y_i and z by constants (the image of x can be computed). And let U be the subgroup of automorphisms of the form

$$\begin{cases} x \mapsto x + a\frac{\partial P}{\partial z} + a^2\frac{1}{2!}\frac{\partial^2 P}{\partial z^2} + a^3\frac{1}{3!}\frac{\partial^3 P}{\partial z^3} + \dots \\ y_i \mapsto y_i \\ z \mapsto z + cy_1^{k_1}y_2^{k_2}\dots y_m^{k_m} \end{cases}$$

Then the group of regular automorphisms of the variety is isomorphic to the semi-direct product $G \leftthreetimes U$.

Sergey Gaifullin. Automorphisms of Danielewski varieties. Journal of Algebra (2021), Q1 SJR

Computer algebra skills

In some areas of the laboratory research, computer algebra systems yield deep results. Our researchers incorporate occasionally such tools into their work.

Sagemath, Python (numpy)

- Alexander Perepechko (+ sympy)
- Boris Bilich

Wolfram Mathematica

- Boris Bilich
- Grigory Taroyan

Singular

- Grigory Taroyan

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