Stable maps to Looijenga pairs

A Looijenga pair is a pair (X,D) with X a smooth complex projective surface and D a singular anticanonical divisor in X. I will describe a series of correspondences relating five different classes of string-theory motivated invariants specified by the geometry of (X,D):

* the log Gromov--Witten theory of (X,D),
* the Gromov--Witten theory of X twisted by the sum of the dual line bundles to the irreducible components of D,
* the open Gromov--Witten theory of special Lagrangians in a toric Calabi--Yau 3-fold determined by (X,D)
* the Donaldson--Thomas theory of a symmetric quiver specified by (X,D), and
* a class of BPS invariants considered in different contexts by Klemm--Pandharipande, Ionel--Parker, and Labastida--Marino--Ooguri--Vafa.

I will also show how the problem of computing all these invariants is closed-form solvable. Joint work with P. Bousseau and M. van Garrel.