Аннотации докладов / Abstracts of the talks

- **Alexei Basalaev**

  «Integrable systems associated to rational Hurwitz-Frobenius manifolds»

  Consider the genus zero Hurwitz space. It has the structure of a Dubrovin–Frobenius manifold. It's well known that this Dubrovin–Frobenius manifold structure gives rise to a KP hierarchy if the length of the ramification profile is one. We will discuss the cases of the other ramification profiles connecting them to the other classical integrable systems.

- **Mikhail Bershtein**

  «Cluster Reductions, Mutations, and q-Painlevé Equations»

  We propose an extension of the Goncharov-Kenyon class of cluster integrable systems by their Hamiltonian reductions. In particular, this extension allows us to fill the gap in cluster construction of the q-difference Painlevé equations. We also propose natural equivalences between such reduced Goncharov-Kenyon integrable systems; these equivalences correspond to mutations in another (dual, in some sense) cluster structure.

  Based on joint work with P. Gavrylenko, A. Marshakov, M. Semenyakin.

- **Василий Болбачан**

  «Гипотеза Гончарова и высшие группы Чжоу»

  Для алгебраического многообразия можно определить его высшие группы Чжоу (=мотивные когомологии). Про них можно думать как про аналог сингулярных гомологий в алгебраической ситуации. Гипотеза Гончарова утверждает что эти группы изоморфны когомологиям так называемого полилогарифмического комплекса, который отражает функциональные соотношения для классических полилогарифмов.

  Я хочу рассказать о доказательстве этой гипотезы в некотором новом случае (степень равна мотивному весу минус один). Доказательство заключается в непосредственной работе с алгебраическими циклами. При этом возникает новый комплекс, который гипотетически квази-изоморфен высшим группам Чжоу.

- **Harry Braden**

  «Some new applications of affine Toda Theory»

  I will describe new and not so new applications of affine Toda Theory to the description of su(2) Euclidean BPS monopoles with symmetry. This arose from discussion of Seiberg-Witten theory with Marshakov, Mironov and Morozov some years ago.

- **Александр Буряк**

  «Деформация формулы Дубровина для пары согласованных скобок Пуассона гидродинамического типа»

  В одной из своих работ, посвященных фробениусовым многообразиям, Дубровин придумал формулу для построения пары согласованных скобок Пуассона гидродинамического типа, стартуя с потенциала произвольного фробениусова многообразия. Я приведу обобщение формулы Дубровина, задающее дисперсионную деформацию этой пары скобок, стартуя с определённого локального функционала, который можно рассматривать, как дисперсионную деформацию потенциала фробениусова многообразия. Удивительным образом, нужные локальные функционалы строятся по некоторым геометрическим данным, связанным с пространством модулей стабильных кривых. В качестве примера этой общей конструкции, возникают пары скобок, задающие иерархию Гельфанда-Дикого, расширенную иерархию Тоды, а также другие известные иерархии математической физики. Доклад основан на совместной работе с Паоло Росси.

- **Boris Bychkov**

  «KP integrability in topological recursion»

  It is well known that potentials corresponding to the invariants of the Chekhov–Eynard–Orantin topological recursion usually are logarithms of tau functions of Kadomtsev–Petviashvili hierarchy. I will define what it means that topological recursion invariants are KP integrable and prove that it is always the case if and only if the spectral curve of this topological recursion is of genus 0.

- **Dmitrii Bykov**

  «The deformed $\mathbb{C}P^1$ sigma model in Gross-Neveu formalism»

  I will review the recently established relation between 2D integrable sigma models and Gross-Neveu (GN) models – a class of models with quartic interaction. This framework can incorporate supersymmetric theories as well as deformed (i.e. non-homogeneous) target spaces. As an example of the latter, one can consider the deformed supersymmetric $\mathbb{C}P^1$ model, also known as the 'sausage'. I will show that this geometry naturally arises as the solution of Ricci flow equations of the deformed GN model, which incidentally are equivalent to Nahm's equations from the theory

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of monopoles. The conformal limit of this system is governed by the so-called super-Thirring model, whose correlation functions can be computed explicitly.

- **Sergei Derkachev**

  «Reflection operator and iterative construction of eigenfunctions for noncompact $SL(2, \mathbb{C})$ open spin chains»

  We consider noncompact open $SL(2, \mathbb{C})$ spin chain and construct eigenfunctions of B-element of monodromy matrix for the simplest case of the chain with one site. We prove orthogonality and completeness of constructed eigenfunctions in the case of one site. The reflection operator appearing in this construction can be used to express eigenfunction for n sites in terms of the eigenfunction for $(n-1)$ sites. We outline this iterative construction and its main building blocks – reflection operator and general solution of Yang-Baxter equation.

- **Petr Dunin-Barkowski**

  «Symplectic duality and logarithmic topological recursion»

  We introduce the new concept of *logarithmic topological recursion*, which allows to extend spectral curve topological recursion to spectral curves for which the functions x and y possess logarithmic singularities. This new definition is natural with respect to the so-called $x - y$ duality. Then we show that the so-called symplectic duality can be expressed as a composition of $x - y$ dualities. This allows to obtain a new and uniform proof of (logarithmic) topological recursion for large families of weighted double Hurwitz numbers.

  The talk is based on a joint work with A.Alexandrov, B.Bychkov, M.Kazarian, and S.Shadrin: arXiv:2312.16950 and arXiv:2405.10720.

- **Vladimir Fock**

  «Group cocycles and clusters»

  Loop algebras of simple Lie algebras possess a well known central extensions. The corresponding loop groups inherit such extensions, but the expression of the corresponding cocycle is much more complicated. One such expression belongs to Kubota for SL(2) and Matsumoto for other groups. We will present a Kubota-like formula with elementary proofs (by drawing pictures) using cluster coordinates on the group.

  We'll also discuss some generalizations and application of this cocycle.

- **Alexander Gorsky**

  «Matsuo-Cherednik duality and truncation of the Calogero eigenfunctions»

  We use the Matsuo-Cherednik duality between the solutions to KZ equations and eigenfunctions of Calogero-Moser Hamiltonians to get the polynomial p^s-truncation of the Calogero-Moser eigenfunctions at a rational coupling constant. The truncation

procedure uses the integral representation for the hypergeometric solutions to KZ equations. The $s \to \infty$ limit to the pure p-adic case has been analyzed in the $n = 2$ case.

- **Gerard Helminck**

  «*Connecting KP and strict KP with their discrete versions*»

  In this talk we describe how the wave matrices of the discrete KP hierarchy and its strict version relate to respectively the wave functions of the KP hierarchy and those of its strict version.

  Two applications of this connection will be given.

- **Alexei Ilin**

  «*Moduli space of cactus flower curves and Gaudin models*»

  I will explain the construction of a specific degeneration of Deligne-Mumford spaces called the moduli space of cactus flower curves. I will also briefly discuss its relationship with trigonometric and (inhomogeneous) rational Gaudin models.

- **Vladimir Kazakov**

  «*Structure constants of huge 1/2-BPS operators of $N = 4$ SYM: matrix model and Riemann-Hopf fluid dynamics*»

  I will describe a new matrix model representation for the protected structure constants of arbitrary 1/2-BPS operators in $N = 4$ SYM. For the "huge" operators, with dimensions of the order $(Nc)^2$, the problem can be reformulated in terms of the integrable Riemann-Hopf fluid dynamics and the underlying algebraic curves. I will present new explicit solutions for particular, representative operators. The talk is based on my paper with H.Murali and P.Vieira. arXiv:2406.01798.

- **Sergei Khoroshkin**

  «*Ruijsenaars system: symmetries and spectral transform*»

  Recent success in the study of Baxter Q operators in Ruijsenaars hyperbolic system led to discovery, besides of bispectral duality, of the duality statements concerning reflection of the coupling constant. It also gives a way to prove orthogonality and completeness of the wave functions. The orthogonality statements can be also stated in nonunitary regimes, where we are able to prove an inversion theorem, which can be regarded as multidimensional generalization of inversion theorem for Fourier and Laplace transforms.

- **Никита Некрасов**

  «*Тау функция Кричевера и калибровочные теории аффинного типа*»

Много лет назад нами с А. Маршаковым и А. Лосевым было подмечено соответствие суперсимметричной статсуммы абелевой калибровочной теории (производящей функции интегралов полиномов от классов Черна тавтологического расслоения над схемой Гильберта точек на $\mathbb{C}^2$) и стационарным сектором теории Громова-Виттена $\mathbb{C}P^1$ (впоследствии выросшее в GW/DT соответствие). Я расскажу о новых результатах, полученных в этом направлении в последние годы (совместно с А.Грековым и И.Кричевером), в частности о препотенциале $N = 2$ теории со старшими временами, обобщающим теорию Вершика-Керова.

- **Marina Nenasheva**

  «Isoperiodic foliation on the space of meromorphic real-normalized differentials»

  We study spaces of meromorphic differentials on complex curves, such that all their periods are real. Such differentials are referred to as real-normalized, they were introduced in the works of I.Krichever. The spaces of real-normalized differentials with the prescribed set of poles and residues at them may be stratified by the orders of the zeroes of the differentials. Differentials with the prescribed group of periods compose isoperiodic subspaces, respecting such stratification. In this talk I plan to describe isoperiodic subspaces in the spaces of real-normalized differentials with a single pole of order two and with two simple poles.

- **Aleksander Orlov**

  «Instanton sum of Fateev–Frolov–Schwartz as a tau function. Massive fermions»

  We shall see that the instanton sum of Fateev-Frolov-Schwartz is a 2KP tau function, however a singular one. For the regularization we relate massive and massless fermions and present various correlation functions as determinants of Bessel functions. We write down bilinear equation for the massive fermions.

- **Gaiane Panina**

  «Diagonal complexes»

  The talk is based on a joint paper with J. Gordon.

  It is known that the partially ordered set of all tuples of pairwise non-intersecting diagonals in an n-gon is isomorphic to the face lattice of a convex polytope called the *associahedron*. We replace the n-gon (viewed as a disc with marked points on the boundary) by an arbitrary oriented surface with a set of labelled marked points ('vertices'). After appropriate definitions we arrive at a cell complex $\mathcal{D}$ (generalizing the associahedron) with the barycentric subdivision $\mathcal{BD}$.

  When the surface is closed, the complex is homotopy equivalent to the space of metric ribbon graphs $RG_{g,n}^{met}$, or, equivalently, to the decorated moduli space $\widetilde{\mathcal{M}}_{g,n}$.

  For bordered surfaces we prove the following.

1) Contraction of an edge does not change the homotopy type of the complex.

2) Contraction of a boundary component to a new marked point yields a forgetful map between two diagonal complexes which is homotopy equivalent to the Kontsevich tautological circle bundle. Thus we obtain a natural simplicial model for the tautological bundle. As an application, we compute the *psi-class*, that is, the first Chern class in combinatorial terms. This result is obtained by using a local combinatorial formula.

3) In the same way, contraction of several boundary components corresponds to the Whitney sum of tautological bundles.

- **Alexander Popolitov**

  «Superintegrability of (q, t)-deformed matrix models in the framework of DIM algebra»

  (q, t)-deformed matrix models are a peculiear generalization of the matrix integrals, with many connections to other branches of modern mathematical physics: to Macdonald polynomials on the algebraic side, and to localization calculations in supersymmetric gauge theories, on the physical side. In my talk I will review the recently emerged approach to discovery of superintegrability formulas in these models, as well as construction of related (generalized) integrable systems, which utilizes the intricate structure of DIM algebra.

- **Vadim Prokofev**

  «Large BKP, B-Toda, Coupled KP hierarchies different approaches»

  We will discuss hierarchy of KP type, which was rediscovered several times in different approaches. It is known by the names of Large BKP, D'_∞, coupled KP, and B-Toda hierarchies. We will give different descriptions of this hierarchy in terms of free fermions, and tau functions.

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- **Vladimir Roubtsov**

  «Higher Bessel functions. From periods to combinatorics»

  We discuss some geometric and combinatorial aspects of the analytic solution to N-quantum differential equation for P^{N-1}.

  Based on the joint paper with Ilia Gaiur and Duco van Straten (arXiv:2405.03015).

- **Андрей Семенов**

  «Слипание инстантонов и эффект увлечения в джозефсоновских контактах»

  В докладе рассматривается простая система из двух джозефсоновских контактов. Динамика этой системы в одном из предельных случаев эквивалентна задаче о случайном блуждании частицы в двумерном периодическом потенциале. В квантовом случае это соответствует последовательному туннелированию между минимумами потенциала и описывается различными многоинстантонными траекториями.

  Было обнаружено, что в такой системе некоторые инстантоны могут объединяться в связанные пары, что приводит к возникновению нового физического явления — эффекта увлечения. В докладе будет показано, что необычного в данном явлении и предложены подходы к его теоретическому описанию.

- **Alexandra Skripchenko**

  «How to generalize interval exchange transformations?»

  Interval exchange transformations can be seen as a very simple combinatorial object: a piecewise linear map of the half-interval that is just a translation on each interval of continuity. These maps play very important role in dynamical systems due to their connection with measured foliations on surfaces and billiards in rational polygons. So the ergodic and combinatorial properties of IETs were widely studied and are quite well understood.

  In my talk I will outline certain generalizations of IETs and possible extensions of the results known for IETs to these maps.

- **Alexey Sleptsov**

  «Vogel's algebra, kernel of sl_N weight system and implications for knot invariants»

  The Vogel's algebra Λ is a 3-valent connected diagram modulo AS and IHX relations with vertex product. Closed Jacobi diagrams are Λ- modules. Pierre Vogel, using character theory, was able to determine the elements of this algebra Λ, which vanish on weight systems of simple Lie algebras. Using such elements, one can find Jacobi diagrams that are in the kernel of the weight system of a given simple Lie algebra. We discuss this construction in the case of the sl_N weight system, and also discuss possible consequences for the Vassiliev invariants and the corresponding quantum knot invariants (colored HOMFLY-PT polynomials).

- **Alexei Yung**

  «Flowing between string vacua for the critical non-Abelian vortex with deformation of $N = 2$ Liouville theory»

  It has been shown that non-Abelian solitonic vortex string supported in four-dimensional (4D) $N = 2$ supersymmetric QCD (SQCD) with the $U(2)$ gauge group and $N_f = 4$

quark flavors becomes a critical superstring. This string propagates in the ten-dimensional space formed by a product of the flat 4D space and an internal space given by a Calabi-Yau noncompact threefold, namely, the conifold. The spectrum of low lying closed string states was found and interpreted as a spectrum of hadrons in 4D $N = 2$ SQCD. In particular, the lowest string state appears to be a massless BPS baryon associated with the deformation of the complex structure modulus b of the conifold. It was recently shown that the Coulomb branch of the associated string sigma model which opens up at strong coupling can be described by $N = 2$ Liouville theory. Building on these results we switch on quark masses in 4D $N = 2$ SQCD and study the interpolation of the initial U(2) SQCD with $N_f = 4$ quarks to the final SQCD with the $U(4)$ gauge group and $N_f = 8$ quarks. To find the true string vacuum which arises due to the mass deformation we solve the effective supergravity equations of motion associated with the deformed world sheet Liouville theory. We show that the massless BPS baryon b survives the deformation and that finding of the spectrum of low lying massive hadrons in the final SQCD is linked to the Calogero problem.

- **Anton Zabrodin**

  «*Multicomponent DKP hierarchy and its dispersionless limit*»

  Using the free fermions technique and bosonization rules we introduce the multicomponent DKP hierarchy as a generating bilinear integral equation for the tau-function. Bilinear equations of the Hirota-Miwa type are corollaries of it. We also consider the dispersionless version of the hierarchy as a set of nonlinear differential equations for the dispersionless limit of logarithm of the tau-function (the F-function). We show that there is an elliptic curve built in the structure of the hierarchy, with the elliptic modulus being a dynamical variable. This curve can be uniformized by elliptic functions, and in the elliptic parametrization many dispersionless equations of the Hirota-Miwa type become equivalent to a single equation having a nice form. This is the joint work with A.Savchenko.

- **Peter Zograf**

  «*Asymptotics of intersection numbers on moduli spaces of curves*»

  Intersection numbers of tautological classes on moduli spaces of n-pointed genus g complex algebraic curves appear in various areas of mathematics and theoretical physics. We will survey the large g and n behavior of these intersection numbers and discuss some of applications.

- **Andrei Zotov**

  «*Relativistic interacting tops at classical and quantum levels and related q-deformed long-range spin chains*»

  First, we review classical elliptic integrable systems of Calogero-Ruijsenaars family and their spin generalizations. The most general is the $GL(NM)$ model of relativistic

interacting tops, which turns into the spin Ruijsenaars-Schneider model when $N = 1$, and turns into $GL(N)$ relativistic Euler-Arnold top in the case $M = 1$. Next, we describe construction of quantum commuting Hamiltonians for the model of interacting relativistic tops. Finally, we explain that the Polychronakos freezing trick being applied to the Hamiltonians provides a new family of q-deformed long-range spin chains. As a simple example this family contains q-deformation of the Haldane-Shastry model introduced by Uglov.